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Analysis of Postbuckling Behavior of Angle-Ply Laminated Plates with Small Initial Curvature under Biaxial Compressive Loads

by

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Abstract

Advanced fiber-reinforced laminated plates have been used as structures in various fields because of their high specific strength and stiffness. In this paper, the postbuckling behavior of angle-ply laminated plates, which are simply supported along four edges, during initial deflection under biaxial compression is considered by the use of Galerkin's method. The inevitability of postbuckling is proved analytically, and the effects of various factors, such as the type of initial imperfection, lamination angle, biaxial compressive load ratio, and the postbuckling deflection pattern, are clarified.

Keywords: *Structural analysis, Composite materials, CFRP, Postbuckling behavior, Angle-ply laminated plates, Biaxial compression, Lamination constitution, Initial imperfection*

1. Introduction

Recently laminated plates are an important structural element in various engineering application. Especially, in aerospace applications where savings are of great importance, advanced fiber-reinforced composite materials are widely used for laminated fiber-reinforced plates and other structural shapes. The composite materials have high in-plane strength but low density. Hence, high strength and high stiffness-to-weight ratios are readily obtained. It is well-known phenomenon that post-buckling of thin plates is stable and hence it remains structurally useful well beyond its primary Euler buckling loads. Post-buckling behaviors of thin laminated plates under uniaxial compression have been discussed by many researchers(1). However, little research has been performed on the secondary buckling phenomenon for thin laminated plates which occurs with further increase of load(2)~(6).

This paper deals with post-buckling problems of orthotropic angle-ply laminated plates with initial small curvature subjected to in-plane biaxial compression, which are simply supported along four edges. A theoretical analysis (7) is used to predict the inevitable post-buckling behavior of angle-ply laminated plates by deflected patterns and initial displacement patterns. Numerical solutions are obtained for the average axial strain, and biaxial compression ratio of the square plate, initial imperfection which are illustrated graphically and discussed.

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2. Fundamental Equations

2.1 Basic Equation

The coordinate system and dimensions of a square plate under biaxial compression are shown in Fig.1. There are u, v and w denote the displacement components in x, y and z directions respectively at the middle surface and w_0 denotes the initial deflection. The in-plane nonliner strain components, ε_x, ε_y and γ_{xy} and the changes of curvatures κ_x, κ_y and κ_{xy} can be written in terms of displacement components as follows.

$$\left.\begin{aligned}&\varepsilon_x=\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x},\quad \varepsilon_y=\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\frac{\partial w}{\partial y}\frac{\partial w_0}{\partial y}\\&\gamma_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}+\frac{\partial w}{\partial x}\frac{\partial w_0}{\partial y}+\frac{\partial w}{\partial y}\frac{\partial w_0}{\partial x}\end{aligned}\right\} \tag{1}$$

$$\kappa_x=-\frac{\partial^2 w}{\partial x^2},\ \kappa_y=-\frac{\partial^2 w}{\partial y^2},\ \kappa_{xy}=-2\frac{\partial^2 w}{\partial x\partial y} \tag{2}$$

The axial, circumferential and shear in the median surface are given in terms of the median surface stresses by the usual form of Hooke's law for thin plates.

$$\left.\begin{aligned}&\varepsilon_x=\frac{\sigma_x}{E_x}-\upsilon_y\frac{\sigma_y}{E_y},\ \varepsilon_y=\frac{\sigma_y}{E_y}-\upsilon_x\frac{\sigma_x}{E_x}\\&\gamma_{xy}=\frac{\tau_{xy}}{G_{xy}}\end{aligned}\right\} \tag{3}$$

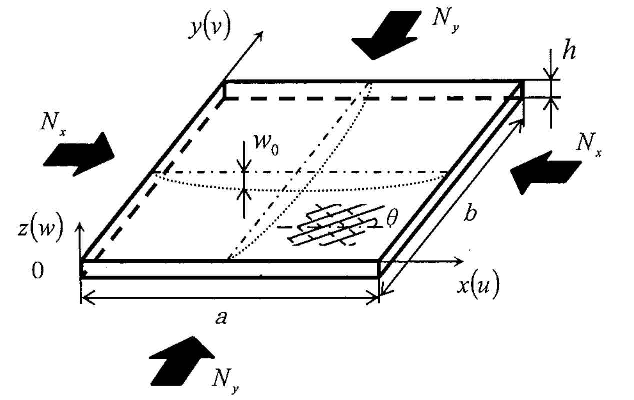


Fig.1 Configuration and coordinates of angle-ply laminated plate with an initial small deflection under biaxial compressive loads

where E_x and E_y are modulus of elasticity, υ_x and υ_y are the Poissons's ratio, and G_{xy} is the shear modulus. The membrane forces per unit width N_x, N_y and N_{xy} are defined as follows:

$$N_x = \int_{-h/2}^{+h/2} \sigma_x dz, \quad N_y = \int_{-h/2}^{+h/2} \sigma_y dz, \quad N_{xy} = \int_{-h/2}^{+h/2} \tau_{xy} dz \tag{4}$$

The membrane force and bending moment for an orthotropic angle-ply laminate are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{6}$$

where A_{ij} are the extensional stiffness and D_{ij} are bending stiffness of laminated plate, A_{ij} and D_{ij} constitute constants of unidirectional composites E_L, E_T, υ_L, υ_T, G_{LT}, and lamination angle θ deg.[8].

2.2 Equilibrium and Compatibility Equations

The equilibrium equations in the x, y and z directions of a plate with an initial imperfection are expressed as follows:

$$\left.\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\ &\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \end{aligned}\right\} \tag{7}$$

$$\begin{aligned} &\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2} + N_x \frac{\partial^2 (w+w_0)}{\partial x^2} \\ &+ 2N_{xy} \frac{\partial^2 (w+w_0)}{\partial x \partial y} + N_y \frac{\partial^2 (w+w_0)}{\partial y^2} = 0 \end{aligned} \tag{8}$$

The Airy function F which satisfied Eq.(7) is defined by

$$N_x = h\frac{\partial^2 F}{\partial y^2}, N_y = h\frac{\partial^2 F}{\partial x^2}, N_{xy} = -h\frac{\partial^2 F}{\partial x \partial y} \tag{9}$$

and reduces the equilibrium equation to the laterial direction to the form of

$$\begin{aligned} &D_{11}\frac{\partial^4 w}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w}{\partial y^4} \\ &= h\left\{ \frac{\partial^2 F}{\partial y^2}\frac{\partial^2 (w+w_0)}{\partial x^2} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 (w+w_0)}{\partial y^2} \right. \\ &\quad \left. - 2\frac{\partial^2 F}{\partial x \partial y}\frac{\partial^2 (w+w_0)}{\partial x \partial y} \right\} \end{aligned} \tag{10}$$

The compatibility equation is given as

$$\begin{aligned} &\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} - \frac{\partial^2 \gamma_{xy}}{\partial x \partial y} \\ &= \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2 - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} + 2\frac{\partial^2 w}{\partial x \partial y}\frac{\partial^2 w_0}{\partial x \partial y} \\ &\quad - \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w_0}{\partial y^2} - \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w_0}{\partial x^2} \end{aligned} \tag{11}$$

The average axial strain of x-axis direction ε_{mx} in the middle of the surface of the plate is defined by

$$\begin{aligned} \varepsilon_{mx} &= -\frac{1}{a}\int_0^a \left(\frac{\partial u}{\partial x}\right) dx \\ &= -\frac{1}{a}\int_0^a \left\{ \varepsilon_x - \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - \frac{\partial w}{\partial x}\frac{\partial w_0}{\partial x} \right\} dx \end{aligned} \tag{12}$$

from which the relation between the non-dimensional average compression K ($=\sigma_x b^2/E_T h^2$, where $\sigma_x = N_x/h$) and the average axial strain of x-axis direction ε_{mx} can be obtained.

Post buckling behavior can then be studied through the solution of non-linear simultaneous Eqs.(10) and (11) involving F and w under the given boundary conditions. Approximate solutions to these equations will be sought since it is very difficult to obtain the exact solutions.

2.3 Analysis of Post-Buckling Behaviors

The in-plane strain and the membrane force are expressed as follows from Eq. (4),

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} & 0 \\ H_{12} & H_{22} & 0 \\ 0 & 0 & H_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \tag{13}$$

where compliance stiffness H_{ij} (i,j=1,2,6) is expressed as follows:

$$\left.\begin{aligned} &H_{11}=A_{22}A_{66}/H, H_{12}=-A_{12}A_{66}/H, H_{22}=A_{11}A_{66}/H, \\ &H_{66}=(A_{11}A_{22}-A_{12}^2)/H, \\ &H=A_{11}A_{22}A_{66}-A_{12}^2A_{66} \end{aligned}\right\} \tag{14}$$

Equilibrium states after primary buckling. The simply supported out-plane boundary conditions are

$$\left.\begin{array}{l} w=\dfrac{\partial^2 w}{\partial x^2}=0 \quad \text{at} \quad x=0, a \\ w=\dfrac{\partial^2 w}{\partial x^2}=0 \quad \text{at} \quad y=0, b \end{array}\right\} \tag{15}$$

and the in-plane boundary conditions are

$$\left.\begin{array}{l} u\text{=constant, along } y\text{-axis,} \quad \left(\int_0^b N_x dy=-P_x\right), \\ \qquad N_{xy}=0 \text{ at } x=0, a \\ v\text{=constant, along } x\text{-axis,} \quad \left(\int_0^a N_y dx=-P_y\right), \\ \qquad N_{xy}=0 \text{ at } y=0, b \end{array}\right\} \tag{16}$$

Here, w and w_0 are expressed by the following two terms which correspond to symmetrical and anti-symmetrical modes along the loading axis,

$$\left.\begin{array}{l} w=w_{11}\sin\dfrac{\pi x}{a}\sin\dfrac{\pi y}{b}+w_{21}\sin\dfrac{2\pi x}{a}\sin\dfrac{\pi y}{b} \\ w_0=c_{11}\sin\dfrac{\pi x}{a}\sin\dfrac{\pi y}{b}+c_{21}\sin\dfrac{2\pi x}{a}\sin\dfrac{\pi y}{b} \end{array}\right\} \tag{17}$$

where c_{11} and c_{21} represent deflection of initial imperfection. The introduction of Eq.(17) into the right-hand side of Eq.(11) yields:

$$\begin{aligned} F=&\frac{1}{2h\lambda^2}(w_{11}{}^2+2w_{11}c_{11}) \left\{\frac{\lambda^4}{16H_{22}}\cos\left(\frac{2\pi x}{a}\right)+\frac{1}{16H_{11}}\cos\left(\frac{2\pi y}{b}\right)\right\} \\ &+\frac{2}{h\lambda^2}(w_{21}{}^2+2w_{21}c_{21}) \left\{\frac{\lambda^4}{256H_{22}}\cos\left(\frac{4\pi x}{a}\right)+\frac{1}{16H_{11}}\cos\left(\frac{2\pi y}{b}\right)\right\} \\ &-\frac{1}{4h\lambda^2}(w_{11}w_{21}+w_{11}c_{21}+w_{21}c_{11}) \left[\frac{\lambda^4}{H_{22}}\left\{\frac{1}{9}\cos\left(\frac{3\pi x}{a}\right)-\cos\left(\frac{\pi y}{a}\right)\right\}\right. \\ &-\frac{\lambda^4\cos(3\pi x/a)\cos(2\pi y/b)}{81H_{22}+36(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+ \\ &\left.\frac{\lambda^4 9\cos(\pi x/a)\cos(2\pi y/b)}{H_{22}+4(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}\right]-\frac{\sigma_x y^2}{2}-\frac{\sigma_y x^2}{2} \end{aligned} \tag{18}$$

where $\lambda=a/b$

When we substitute deflection w and stress function F for the equilibrium equation in the lateral direction, i.e., as Eq.(10), the following simultaneous cubic equation is obtained by applying the Galerkin method:

$$\begin{aligned} &\frac{\pi^2}{4}\left(\frac{w_{11}}{a^2\lambda}\right)\left\{D_{11}+2(D_{12}+2D_{66})\lambda^2+D_{22}\lambda^4\right\} \\ &+\frac{\pi^4}{64}\left(\frac{1}{ab}\right) \\ &\times\left[\begin{array}{l} \dfrac{1}{H_{11}\lambda^2}(w_{11}+c_{11})\left\{(w_{11}{}^2+2w_{11}c_{11})+4(w_{21}{}^2+2w_{21}c_{21})\right\} \\ +\dfrac{1}{H_{22}}\lambda^2(w_{11}+c_{11})(w_{11}{}^2+2w_{11}c_{11}) \\ +\lambda^2(w_{21}+c_{21})(w_{11}w_{21}+w_{11}c_{21}+w_{21}c_{11}) \\ \times\left[\dfrac{81}{H_{22}+4(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+\dfrac{1}{81H_{22}+36(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+\dfrac{4}{H_{22}}\right] \end{array}\right] \\ &\times\left[\begin{array}{l} \left(\dfrac{8}{3}\right)w_{11}(w_{11}+c_{11})\dfrac{1}{H_{22}+(2H_{12}+H_{66})\lambda^2+H_{11}\lambda^4} \\ +\left(\dfrac{128}{5}\right)w_{21}(w_{21}+c_{21})\dfrac{1}{16H_{22}+4(2H_{12}+H_{66})\lambda^2+H_{11}\lambda^4} \\ +\dfrac{1}{H_{22}}\left\{\left(\dfrac{1}{6}\right)(w_{11}{}^2+2w_{11}c_{11})+\left(\dfrac{1}{30}\right)(w_{21}{}^2+2w_{21}c_{21})\right\} \end{array}\right] \\ &-\sigma_x h\frac{\pi^2}{4\lambda}(w_{11}+c_{11})(1+\lambda^2 k_y)=0 \end{aligned} \tag{19}$$

$$\begin{aligned} &\frac{\pi^4}{4}\left(\frac{w_{21}}{a^2\lambda}\right)\left\{16D_{11}+8(D_{12}+2D_{66})\lambda^2+D_{22}\lambda^4\right\} \\ &+\frac{\pi^4}{64}\left(\frac{\lambda}{b^2}\right) \\ &\times\left[\begin{array}{l} \dfrac{4}{H_{11}\lambda^4}(w_{21}+c_{21})\left\{(w_{11}{}^2+2w_{11}c_{11})+4(w_{21}{}^2+2w_{21}c_{21})\right\} \\ +\dfrac{1}{H_{22}}(w_{21}+c_{21})(w_{21}{}^2+2w_{21}c_{21}) \\ +(w_{11}+c_{11})(w_{11}w_{21}+w_{11}c_{21}+w_{21}c_{11}) \\ \times\left[\dfrac{81}{H_{22}+4(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+\dfrac{1}{81H_{22}+36(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+\dfrac{4}{H_{22}}\right] \end{array}\right] \\ &\times\left[\begin{array}{l} \left(\dfrac{128}{5}\right)w_{21}(w_{11}+c_{11})\dfrac{1}{16H_{22}+4(2H_{12}+H_{66})\lambda^2+H_{11}\lambda^4} \\ +\left(\dfrac{32}{5}\right)w_{11}(w_{21}+c_{21})\dfrac{1}{H_{22}+(2H_{12}+H_{66})\lambda^2+H_{11}\lambda^4} \\ +(w_{11}w_{21}+w_{11}c_{21}+w_{21}c_{11}) \\ \times\left\{\dfrac{2}{H_{22}+4(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+\left(\dfrac{1}{5}\right)\dfrac{6}{81H_{22}+36(2H_{12}+H_{66})\lambda^2+16H_{11}\lambda^4}+\dfrac{16}{15H_{22}}\right\} \end{array}\right] \\ &-\sigma_x h\frac{\pi^2}{\lambda}(w_{21}+c_{21})\left(1+\frac{\lambda^2 k_y}{4}\right)=0 \end{aligned} \tag{20}$$

where, k_y $(=\sigma_y/\sigma_x)$ is biaxial compressive load ratio.

Then w_{11} and w_{21} are determined by applying the Newton-Raphson method to equation(19) and (20). Substituting w and w_0 into equation(12), the relationship between the average axial compressive stress σ_x and the average axial shortening ε_{mx}, is obtained as follows:

$$\varepsilon_{mx} = (H_{11} + H_{12}k_y)\sigma_x h + \frac{\pi^2}{8a^2}(w_{11}{}^2 + 2w_{11}c_{11} + 4w_{21}{}^2 + 8w_{21}c_{21}) \qquad (21)$$

3. Numerical Results and Discussion

The post-buckling behavior of a square angle-ply laminated plate with initial deflection, which is simply supported on four edges under biaxial compression, is analyzed by the procedure mentioned above. The effects of various factors on the postbuckling stresses are discussed herein.

The elastic constants of unidirectional carbon fiber-reinforced epoxy composites with fiber volume content V_f = 60% used for numerical example are as follows.

$$\left.\begin{array}{l} E_L = 137\ \text{(GPa)},\quad E_T = 8.18\ \text{(GPa)},\ G_{LT} = 4.75\ \text{(GPa)} \\ \nu_L = 0.316,\quad \nu_T = 0.0189 \end{array}\right\} \qquad (22)$$

These values can be obtained by explicit algebraic equations when the constitutent materials and V_f are specified, and they are confirmed by experiments [9].

The method of solution outlined in the foregoing will be applied to investigate the post-buckling behavior of square angle-ply laminated plates.

In that follows, numerical solutions will be obtained of initially flat plates and plates with initial deflection, for example c_{11} the magnitude of which is defined as 0.05 or 0.10 h.

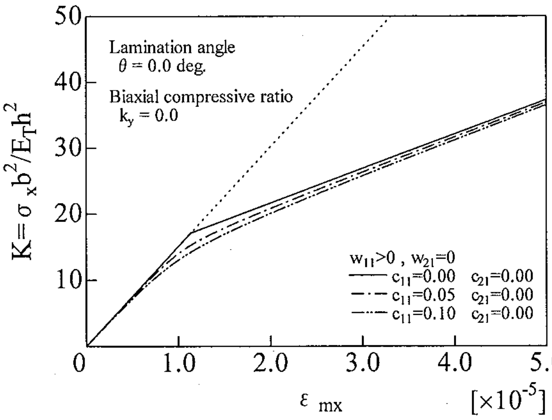
The relationship between non-dimensional average axial compression $K(=\sigma_x b^2/E_T h^2)$ and average axial strain of x-axis direction ε_{mx} is shown in Fig.2 to Fig.4, where $k_y(=N_y/N_x)$ is biaxial compressive load ratios.

Figure2 shows the relationship between K and ε_{mx} at λ $(=a/b)$=1.0, when k_y=0.0, that is, the biaxial compressive ratios corresponding is the uniaxial compression. And, the case of θ =0, 45, 90 deg. with initial deflection c_{11}=0.05 and 0.10, c_{21}=0.05 and 0.10 are shown in the form of non-dimensional average axial compression K and average axial strain of x-axis direction ε_{mx} in Fig.2 (a), (b) and (c), respectively.
Moreover, Fig. 3 and Fig.4 show K and ε_{mx} at λ $(=a/b)$=1.0, when the biaxial compressive ratios k_y are 0.5 and 1.0, respectively.

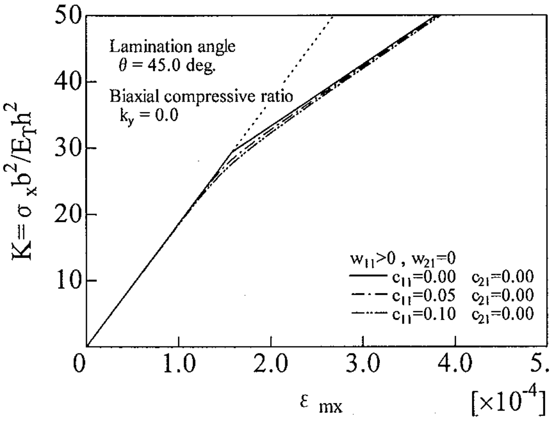
Non-imperfection c_{11}=0.0 and c_{21}=0.0 are illustrated by solid line, and c_{11} or c_{21}=0.05 is illustrated by dash-dotted line, c_{11} or c_{21}=0.10 is illustrated by chain double-dotted line in every figure.

As shown from Fig.2 to Fig.4, this curve becomes a straight line of equal slope prior to primary buckling without reference to initial deflections. After buckling the slope of this line may decrease. It indicates a loss of stiffness in the postbuckling range and is greatly dependent upon the number of half-waves of x and y direction.

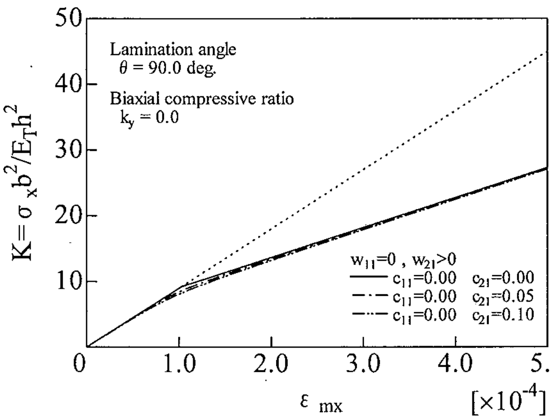
As for effect of lamination angle θ deg., the maximum slope of postbuckling is indicated at θ =45 deg., and the slope decreases with



(a) lamination angle θ =0 deg.



(b) lamination angle θ =45 deg.



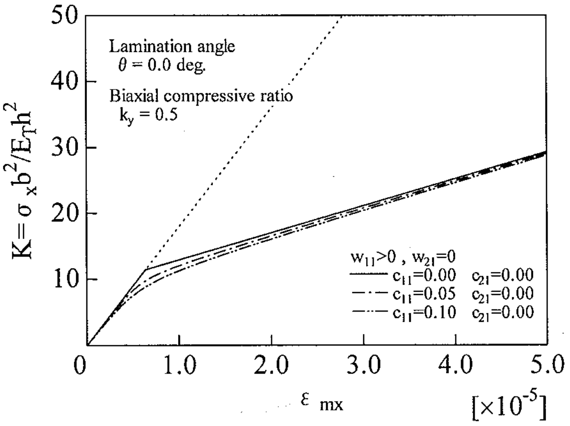
(c) lamination angle θ =90 deg.

Fig.2 The relationship between axial compressive stress and average axial shortening with initial imperfection (k_y=0.0).

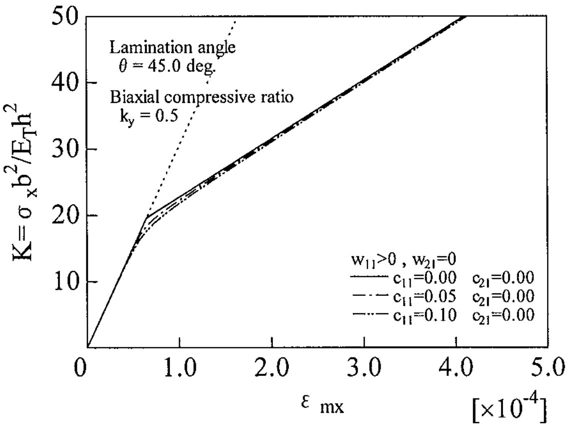
increasing lamination angle θ deg..

As for effect of biaxial compression ratio k_y, it is seen from the Fig.2 to Fig.4 that the postbuckling K decreases with increasing biaxial compression ratio for a square plates.

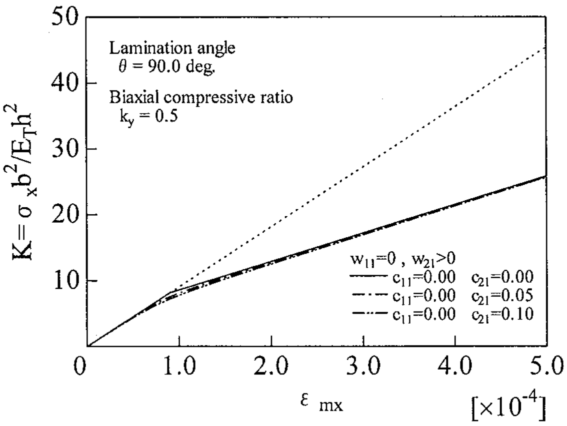
One of authors indicated the postbuckling behaviors of angle-ply laminated plates without initial deflection [10], as for effects of initial deflection, as shown from Fig.2 to Fig.4, the post-buckling K decreased with increasing initial deflection.



(a) lamination angle θ =0 deg.

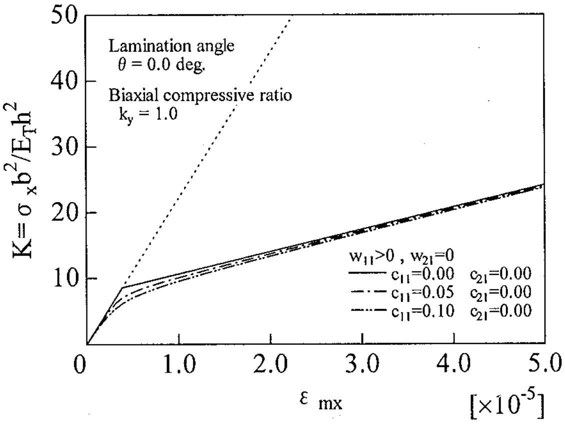


(b) lamination angle θ =45 deg.

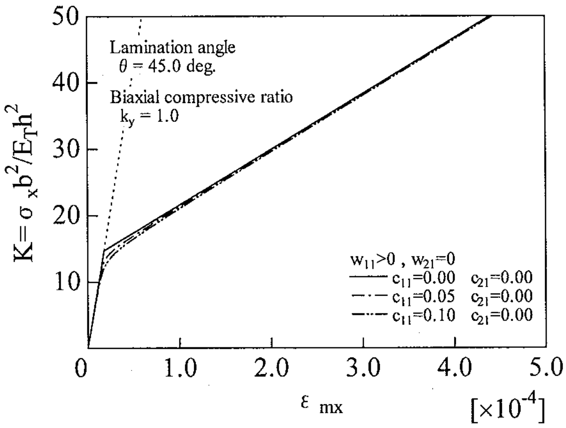


(c) lamination angle θ =90 deg.

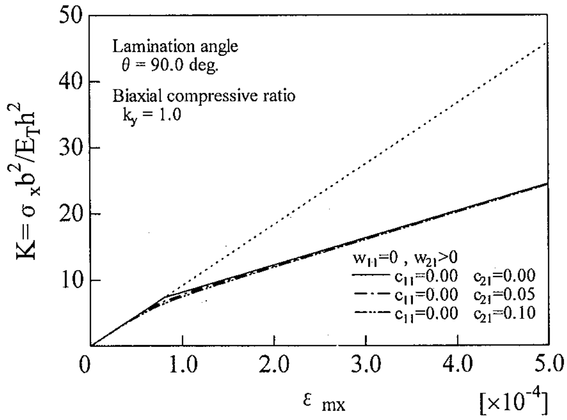
Fig.3 The relationship between axial compressive stress and average axial shortening with initial imperfection (k_y=0.5).



(a) lamination angle θ =0 deg.



(b) lamination angle θ =45 deg.



(c) lamination angle θ =90 deg.

Fig.4 The relationship between axial compressive stress and average axial shortening with initial imperfection (k_y=1.0).

4. Conclusion

It is shown analytically that the postbuckling behavior for simply supported angle-ply laminated plates with initial small curvature can be proved.

The effects of various factors on the postbuckling are discussed with the following conclusions:

a) As for effect of lamination angle θ deg., the maximum slope of postbuckling is indicated at θ =45 deg., and the slope decreases with increasing lamination angle θ deg..

b) As for effect of biaxial compression ratio k_y, the postbuckling K decreases with increasing biaxial compression ratio for a square plates.

c) As for effects of initial deflection, the postbuckling K decreased with increasing initial deflection.

The future direction of this study will be analysis the postbuckling behavior of angle-ply laminated rectangular plates with initial imperfection and secondary buckling of angle-ply laminated square plates with initial imperfection. And we would now like to conduct optimal design for secondary buckling of composite laminated plates.

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