

# Secondary Compression of Anisotropic Consolidated Clays

by

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## Abstract

A constitutive model capable of accounting for time dependency and the quasi-pre-consolidated effect is presented. The model adopts the concept of an anisotropic bounding surface combining, elastoplastic and viscoplastic theory to simulate secondary compression of quasi-pre-consolidated clay under the multidimensional condition. Predictive capability of the proposed model is confirmed by  $K_0$  and anisotropic consolidation tests.

*Keywords: Secondary compression, Quasi-pre-consolidation, Anisotropic consolidation, Elasto-viscoplastic soil model*

## 1. Introduction

The damage to a construction due to long term differential settlement and its restoration are big problems in road embankment on soft ground, and it is important to grasp long term settlement precisely at the designing time. Compared with one-dimensional consolidation in residential land embankment on soft ground, it is said that long term settlement in road embankment under plain strain condition contains the effect of lateral flow.<sup>1)</sup> Although the total settlement under multi-dimensional condition is generally calculated as the sum of the consolidation settlement and the so called immediate settlement, which is the amount of settlement with shearing deformation, there are few studies that consider the time dependency of settlement with shearing deformation.<sup>2), 3)</sup> And, within the range that the authors examined, the secondary compression difference between one dimensional and multi dimensional long term settlement, is not thoroughly studied.<sup>4)</sup> Secondary compression is influenced by many factors.<sup>5), 6), 7)</sup>

The authors consider the consolidation stress condition and stress history. As most of the real soft ground is under quasi-pre-consolidated condition due to long term secondary

compression, it is necessary to account for its influence on the consolidation analysis of soft ground.<sup>8)</sup>

This paper examines the influence of the quasi-pre-consolidation period and anisotropic stress on secondary compression by consolidation test with triaxial test apparatus. Subsequently, the simulation of anisotropic consolidation test using a simple secondary compression model with a minimum number of constants was compared with the experimental result.

## 2. Experimental procedure

Table 1 shows the physical properties of clays considered in this work. Remolded clay sample with water content over the liquid limit for consolidation test was prepared by pre-consolidation in one-dimensional consolidation test apparatus. A clay sample of 5cm diameter and 12cm height was formed, wrapped at the top, bottom and circumference with filter paper in order to allow consolidation, and set in triaxial cell. The sample was pre-consolidated for a day at  $K_0$  consolidation under the pressure that is equal to the pre consolidation pressure, and subsequently the following consolidation tests were done.

### (1) $K_0$ consolidation test

Vertical stress increments were imposed in two steps.

Table 1 Physical properties of clays

Sample	Gs	LL(%)	PL(%)	Grading (%)		
				caly	silt	sand
A	2.64	112.0	50.5	47.0	38.4	14.6
B	2.67	69.0	37.0	24.0	52.0	24.0

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The consolidation periods, after the first step of consolidation pressure (98-245kPa), are varied between 60 minutes (almost equal to the End of Primary Consolidation determined by the square root of time method), one day and one week, and the influence of the consolidation period of the first step on secondary consolidation in the next loading step (245-409 kPa) is examined.

**(2) Anisotropic consolidation test (1)**

After  $K_0$  consolidation with vertical stress of 98 kPa, the clay sample was loaded five times with small stress increments at intervals of 20 minutes to avoid the excessive shearing deformation. At the end of loading, the stress ratio  $\eta$  ( $=q/p$  = the ratio of deviator stress  $q$  and effective mean stress  $p$ ; prime showing effective stress is omitted in this paper) is 1, 1.21, 1.31, 1.4, and a common axial stress  $\sigma_a$  ( $=196$  kPa) is adopted. The relation between  $\eta$  of anisotropically consolidated clay and secondary consolidation was examined.

**(3) Anisotropic consolidation test (2)**

The clay sample was  $K_0$  consolidated with vertical stress of 196 kPa, and while the mean stress was kept constant and deviator stress was increased, the influence of dilatancy on secondary compression was examined. The stress ratio  $\eta$  which is equal to 0.93 for  $K_0$  consolidation, was changed from 1.21, to 1.31, to 1.4 during anisotropic consolidation.

**3. Results and discussions**

**(1)  $K_0$  consolidation test results**

Fig.1 shows the relation of void ratio and elapsed time at the second loading step in  $K_0$  consolidation test. Secondary compression almost in proportion to the logarithm of time, was observed after 60 minutes of consolidation. Different coefficients of secondary compression are observed, and it is thought to be due to the different consolidation periods of the former(or first) loading step. The response at secondary

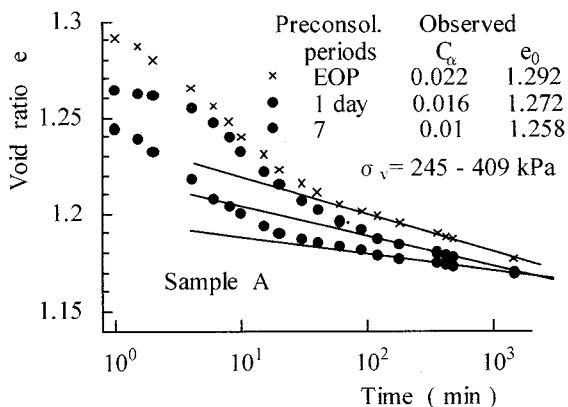


Fig. 1 Observed  $K_0$  consolidation time curves

compression is affected by the initial void ratio at the end of primary consolidation. If the sample at the EOP loading is normally consolidated, the sample with consolidation period after EOP, is a quasi pre consolidated one, and Fig.1 shows the rate decrease during secondary compression due to quasi-pre-consolidation.

**(2) Anisotropic consolidation test results**

Fig.2 and Fig.3 show the relation of volumetric strain, axial strain and elapsed time at anisotropic consolidation test (1). Volumetric strain decreases in proportion to the magnitude of average effective stress at anisotropic consolidation. Volumetric strain and axial strain are in proportion to the logarithm of time after 200 minutes. With the rate of secondary volumetric compression  $\alpha_v$  defined at different  $\eta$  Fig.2 shows that  $\alpha_v$  at  $K_0$  consolidation is the largest and that increase of  $\eta$  causes decrease of  $\alpha_v$ . Fig.3 shows the increase of axial strain due to the increase of  $\eta$ , hence the increase of rate of secondary compression  $\alpha_a$  defined in terms of axial strain, as well. Fig.4 and Fig.5 are the results of anisotropic consolidation test (2). Change of strain with time similar to anisotropic test (1) is observed. As the mean stress was kept constant during the test (2), it is thought that the time dependency of negative dilatancy influences the rate of secondary compression  $\alpha_v$  defined by volumetric strain. Fig.6 shows the relationship between rate of secondary compression and stress ratio at anisotropic consolidation test.

$\alpha_v$  decreases a little due to the increase of  $\eta$ , while  $\alpha_a$  increases a lot. In real soft ground, if  $\alpha_a$  is larger at anisotropic consolidation than at one dimensional consolidation, test results show that there is a possibility that the long term settlement is greater on road embankment than on residential land embankment. Thus, It is necessary to study in detail the time dependency of volumetric strain and shear strain at anisotropic consolidation.

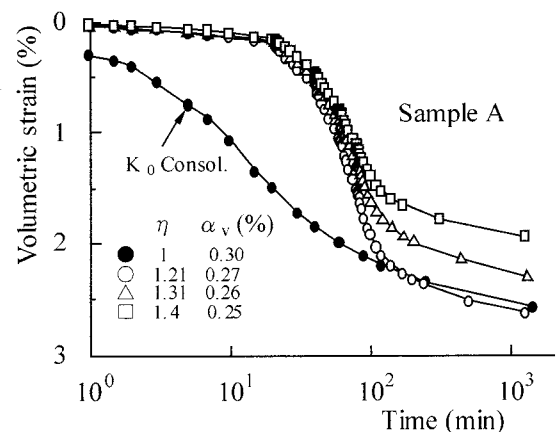


Fig. 2 Observed volumetric strain time curves

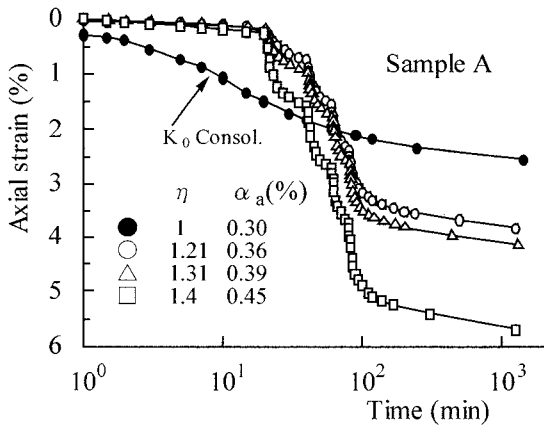


Fig. 3 Observed axial strain time curves

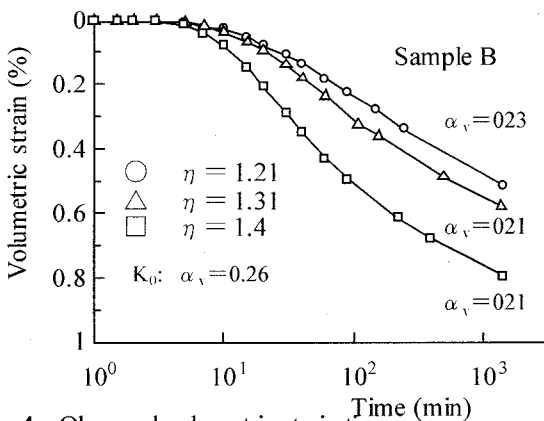


Fig. 4 Observed volumetric strain time curves

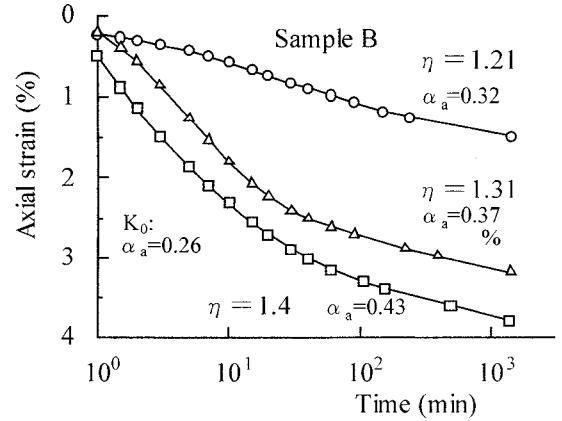


Fig. 5 Observed axial strain time curves

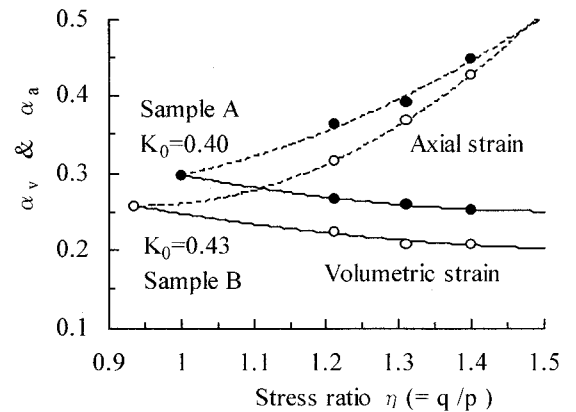


Fig. 6 Relations of stress ratio and coefficient of secondary compression

4. Simulations of consolidation

(1) Stress strain time relation

As the void ratio  $e_A$  of a clay element in  $K_0$  consolidation at point A in Fig.7 decreases to  $e_0$  by secondary compression, the volumetric strain of quasi-pre-consolidated clay is examined.<sup>8), 9), 10)</sup> The  $\lambda$  line of elasto plastic volumetric strain is shown with the dotted line which goes through point a in Fig.7b). The effective mean stress, shown by point b in Fig.7, at the intersection of the  $\lambda$  line and the  $\kappa$  line, the latter passing from  $e_0$ , is considered to be associated with the expansion of the yield surface due to quasi-pre-consolidation. The so expanded yield surface will be considered as Bounding surface within the Bounding surface soil plasticity theory.<sup>11), 12)</sup> The dotted line in Fig.7a) represents the Loading surface passing through point A, which is supposed to be analogous to the Bounding surface shown by the solid line. The range between the Loading surface and Bounding surfaces, is supposed to represent quasi-pre-consolidated range. Strain increments which change in proportion to effective stress, are calculated by using the associated flow rule and the Radial mapping rule, associated with the structure of Bounding surface

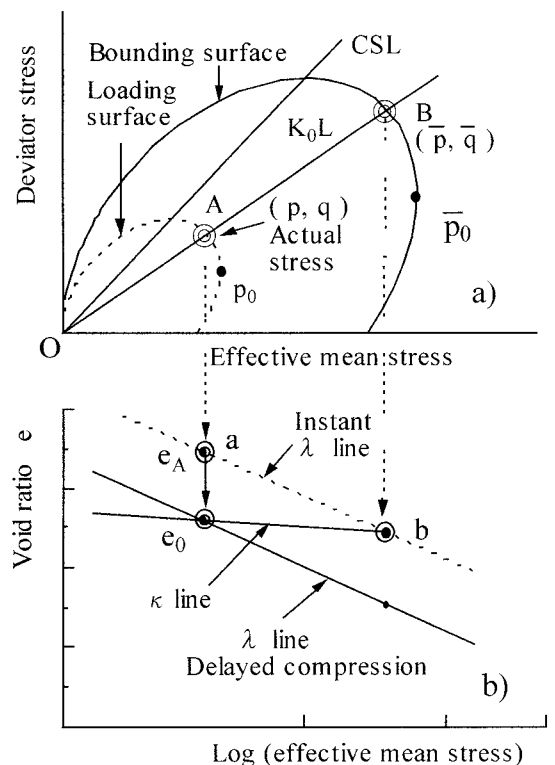


Fig. 7 Bounding surface and  $e \sim \log p$  relations

constitutive modeling.<sup>11), 12)</sup>

The total volumetric strain rate  $\dot{v}$  of a clay element and the total shearing strain  $\dot{\epsilon}$  rate are expressed by Eq.1 and Eq.2 as the sum of elastic, plastic and visco-plastic strain rate components, respectively.

$$\dot{v} = \dot{v}^e + \dot{v}^p + \dot{v}^{vp} \quad (1)$$

$$\dot{\epsilon} = \dot{\epsilon}^e + \dot{\epsilon}^p + \dot{\epsilon}^{vp} \quad (2)$$

where superscripts e, p, and vp are elastic, plastic and visco plastic strain components, respectively.

The sum of elastic and plastic strain rates is obtained by the specific form of Eq.3 – Eq.6 as

$$\begin{aligned} \dot{v}^e + \dot{v}^p = & \left( \frac{1}{H} \frac{\partial F}{\partial \bar{p}} \frac{\partial F}{\partial \bar{p}} + \frac{1}{K} \right) \dot{p} \\ & + \frac{1}{H} \frac{\partial F}{\partial \bar{p}} \frac{\partial F}{\partial \bar{q}} \dot{q} \end{aligned} \quad (3)$$

$$\begin{aligned} \dot{\epsilon}^e + \dot{\epsilon}^p = & \frac{1}{H} \frac{\partial F}{\partial \bar{p}} \frac{\partial F}{\partial \bar{q}} \dot{p} + \\ & \left( \frac{1}{H} \frac{\partial F}{\partial \bar{q}} \frac{\partial F}{\partial \bar{q}} + \frac{1}{3G} \right) \dot{q} \end{aligned} \quad (4)$$

$$H = - \frac{\partial F}{\partial v^p} \frac{\partial F}{\partial \bar{p}} \left( \frac{\bar{r}}{r} \right) \quad (5)$$

$$K = \frac{1+e}{\kappa} p \quad \text{and} \quad G = \frac{3(1-2\nu)}{1+\nu} K \quad (6)$$

where p is the effective mean stress, q is the deviator stress, F is the analytical expression of the Loading surface to be defined subsequently, H is the plastic modulus acting as a hardening parameter, K is the elastic bulk modulus, e is the void ratio,  $\kappa$  is the slope of swelling line,  $\nu$  is the Poisson's ratio,  $\bar{r}$  and r are the distances in stress space shown in Fig.7 as  $\bar{r} = OB$ ,  $r = OA$ , the superposed bar means a stress on the Bounding surface.

The visco plastic strain rates are expressed by Eq.7, in term of an over stress function  $\phi$  which is defined in Eq.8.

$$\dot{v}^{vp} = \langle \phi \rangle \frac{\partial F}{\partial \bar{p}} \quad \text{and} \quad \dot{\epsilon}^{vp} = \langle \phi \rangle \frac{\partial F}{\partial \bar{q}} \quad (7)$$

$$\phi = \alpha_v * 10^{-(v^{vp} + v_0^{vp})/\alpha_v} * \left( r / \bar{r} \right) \quad (8)$$

The  $v_0^{vp}$  is the viscoplastic volumetric strain given as the initial condition at the beginning of calculation. It can be considered that there are changes of secondary volumetric compression rate  $\alpha_v$  which is affected by the ratio of quasi-pre-consolidation. But, because it is impossible to predict the

rate of secondary compression  $\alpha_v^{OC}$  developed in quasi-pre-consolidated clays, over consolidation ratio is substituted for quasi-pre-consolidation ratio. The secondary compression rate of quasi-pre-consolidated clay is calculated by of Eq.9. Superscripts OC and NC mean over consolidation and normally consolidation, respectively.

$$\alpha_v^{OC} = \alpha_v^{NC} * (0.1 + 0.9 / OCR) \quad (9)$$

For the application of Eq.1 and Eq.2, it is important to specify the analytical expression of the Bounding surface and its analogous Loading surface. As such, the anisotropic expression derived by Dafalias(1986) is chosen, given by the following equations.

$$F = M^2(p^2 - pp_0) + q^2 - 2\beta pq + \beta^2 pp_0 \quad (10)$$

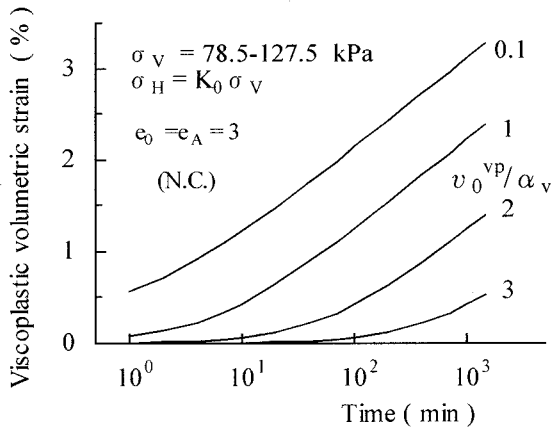
$$\beta = \frac{\eta_K^2 + 3(1 - \kappa / \lambda)\eta_K - M^2}{3(1 - \kappa / \lambda)} \quad (11)$$

where M is the slope of the critical state line in q-p space,  $p_0$  is a size parameter of the loading surface,  $\beta$  is the anisotropic variable which determine the rotation and distortion of the surface shape, and which can be determined under  $K_0$  conditions by Eq.11, where  $\eta_K = 3(1 - K_0)/(1 + 2K_0)$ . If  $\beta = 0$ , F agrees with the isotropic yield function of Modified Cam Clay Model.

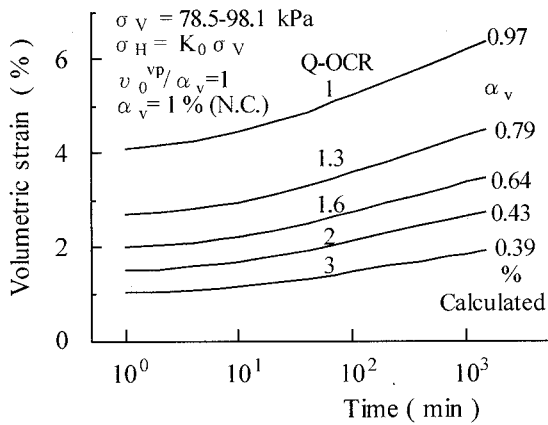
In this paper, the ratio of bounding surface and loading surface  $\bar{p}_0/p_0$  is defined as the quasi-pre-consolidation ratio Q-OCR. In calculating the stress strain relation including secondary compression, the model in this paper requires six soil constant : slope of critical state line M, compression index  $\lambda$ , swelling index  $\kappa$ , coefficient of secondary compression  $\alpha_v$ , coefficient of earth pressure at rest  $K_0$  and Poisson's ratio  $\nu$ . If the initial void ratio of the clay  $e_0 = e_A$  in Fig.7, normally consolidated condition applies. If  $\alpha_v = 0$  or  $\phi = 0$  in Eq.8, elasto plastic constitutive response applies without viscoplasticity.

## (2) Computational results and discussions

The stress strain time relation of soil under drained conditions is calculated using the proposed equations. The constants used in the calculation are shown in Table 2. In order to calculate stress strain time relation, it is necessary to fix  $v_0^{vp}$  in Eq.8 as initial condition at the beginning of anisotropic consolidation. Visco plastic volumetric change in time of normally consolidated clay is calculated, while the ratio between  $v_0^{vp}$  and secondary compression constant  $\alpha_v$



**Fig. 8** Calculated viscoplastic volumetric strain time curves of normally consolidated clay

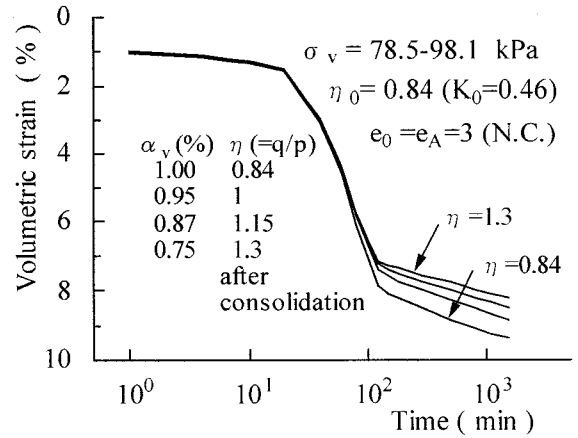


**Fig. 9** Calculated total volumetric strain time curves of quasi pre consolidated clay

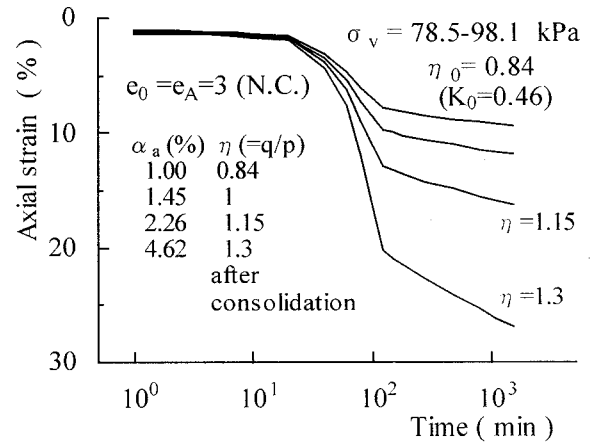
$\lambda$	$\kappa$	M	$K_0$	$\alpha_v$	$\nu$
0.75	0.09	1.35	0.46	1	0.3

is changed with stress increase along  $K_0$  line, and the result is shown in Fig.8. Time increment in the calculation is one minute.

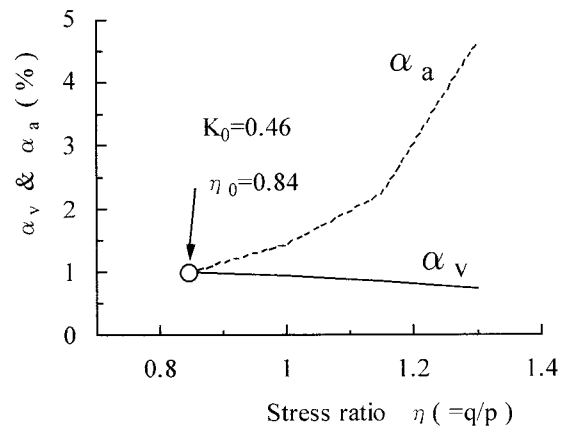
Visco plastic volumetric strain (equivalent to secondary compression) is calculated in proportion to the logarithm of time, after a certain period following consolidation and corresponding with the size of supposed  $v_0^{vp}$ . The smaller  $v_0^{vp}$  is, the more linear the line becomes. However, it is difficult to decide the size of viscoplastic volumetric strain at the beginning of anisotropic consolidation by an experiment. So, in this paper,  $v^{vp}/\alpha_v = 1$  is adopted in hereafter calculations, because the viscoplastic volumetric strain at the beginning of anisotropic consolidation is almost zero and it is comparatively linear. In the anisotropic



**Fig. 10** Calculated volumetric strain time curves



**Fig. 11** Calculated axial strain time curves



**Fig. 12** Calculated relations of stress ratio and coefficient of secondary compression

consolidation with stress increment along the  $K_0$  line, the principal strain and volumetric strain are calculated to be the same.

So change in time of viscoplastic volumetric strain in Fig.8 can be regarded as secondary compression of clay

with infinite coefficient of permeability under the condition of  $K_0$  consolidation. Fig.9 shows the total volumetric strain time curves of quasi-pre-consolidated clay. The total volumetric strain at one minute of elapsed time after the beginning of anisotropic consolidation is almost equal to the elasto plastic volumetric strain. The change in time of the total volumetric strain, calculated with quasi-pre-consolidation ratio shown in the figure, is the same as the secondary compression rate calculated by Eq.10.

The calculated results of change in time of volumetric and axial strain under similar condition of anisotropic consolidation, are shown in Fig.10 and Fig.11. As calculation is made under fully drained condition, it does not reproduce the consolidation process of the anisotropic consolidation test, but each change calculated in time is quite similar to the test results. From calculation of anisotropic consolidation test (2), the relation between the rate of secondary consolidation and stress ratio  $\eta$  is examined, and the result is shown in Fig.12. The relation between secondary compression coefficients  $\alpha_v$  and  $\alpha_a$ , defined by calculated volumetric and axial strain, respectively, and the stress ratio  $\eta$  shows results that are quite similar to the observed ones.

## 5. Conclusion

Secondary compression, which is thought to cause long term settlement, is studied by anisotropic consolidation test and its simulation. The obtained results are summarized as follows.

- 1) As stress ratio increases at anisotropic consolidation, secondary consolidation rate defined by volumetric strain decreases and secondary consolidation rate defined by axial strain increases. The change of secondary compression rate with the increment of stress ratio is considerably larger if the secondary compression rate is defined by the axial strain. The result suggests the possibility that long term settlement under multi-dimensional consolidation condition becomes larger than under one-dimensional consolidation condition.
- 2) Assuming that the bounding surface expands as void ratio decreases with secondary compression, the range between bounding surface and loading surface is defined as quasi pre consolidation range. The simulation of drained shear test of normally and quasi pre consolidated clay by an elasto visco plastic soil model, which is an application of the concept of bounding surface proposed by Dafalias, is similar to secondary compression behavior of anisotropic consolidation test. The proposed model can be used to predict the long term settlement under multi-dimensional condition.

## References

- 1) Takeshima, M.: Long term settlement of embankments on soft grounds, *TSUCHI TO KISO, JSSMFE*, No.253, pp. 37-44, 1979. ( in Japanese )
- 2) Skempton, A.W. and Bjerrum, L.: A contribution to the settlement analysis of foundations on clay, *Geotechnique*, Vol.7, No.4, pp.168-178, 1957.
- 3) Yasuhara, K.: Analysis of deformation behavior of saturated soft clay in anisotropic consolidation, *Proc. of JSCE*, No.283, pp.67-78, 1979. ( in Japanese )
- 4) Yasuhara, K. and Yamanouchi, T.: Deformation characteristics in triaxial consolidation of anisotropic consolidated clay, *Proc. of JSCE*, No.246, pp.93-103, 1976. ( in Japanese )
- 5) Yasukawa, I. And Kamon, M.: Effect of loading conditions on secondary consolidation of cohesive soils, *Soils and Foundations*, Vol.27, No.2, pp.93-106, 1987. ( in Japanese )
- 6) Murakami, Y.: Excess pore water pressure and pre consolidation effect developed in normally consolidated clays of some age, *Soils and Foundations*, Vol.19, No.4, pp.17-29, 1979.
- 7) Akai, K. and Adachi, T.: Study on the one dimensional consolidation and shear strength characteristics of fully saturated clay in terms of effective stress, *Proc. of JSCE*, No.113, pp.11-27, 1965. ( in Japanese )
- 8) Bjerrum, L.: Engineering geology of Norwegian normally consolidated marine clays related to settlements of Buildings, *Geotechnique*, Vol.17, No.2, pp. 81-118, 1967.
- 9) Borja, R.I. and Kavazanjian, E.: A constitutive model for the stress strain time behavior of wet clays, *Geotechnique*, Vol.35, No.3, pp.283-298, 1985.
- 10) Kutter, B.L. and Sathialingam, N.: Elastic viscoplastic modeling of the rate dependent behavior of clays, *Geotechnique*, Vol.42, No.3, pp.427-441, 1992.
- 11) Dafalias, Y.F.: Bounding surface elastoplasticity viscoplasticity for particulate cohesive media, *IUTAM Symp. on Deformation and Failure of Granular Materials*, Delft., pp.97-107, 1982.
- 12) Kaliakin, V.N. and Dafalias, Y.F.: Verification of the elastoplastic viscoplastic bounding surface model for cohesive soils, *Soils and Foundations*, Vol.30, No.3, pp.25-36, 1990.

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