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Effects of Flow on Transmission Loss Characteristics of Silencers of the Multiple Helmholtz Resonator Type

by

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Abstract

This paper describes transmission loss characteristics of silencers composed of multiple Helmholtz resonators. Numerical and experimental results show that the transverse arrangement of resonators can be treated by equally dividing the plane wave front propagated through a flow duct into the cross-sectional area corresponding to individual resonator. It is also shown that the longitudinal arrangement increases the attenuations by these multieffects, even though the performance of each resonator may be lowered by the separated flow close to its entrance.

Keywords: Helmholtz resonators, Transmission loss, Mach number, Resonance frequency, Multieffects.

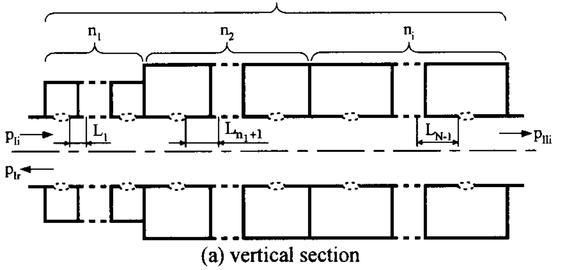
1. *Introduction*

The Helmholtz resonator is widely used as a silencer in duct systems. On the basis of the linear theory, disregarding flow, Davis et al, presented the equations for a single resonator and multiple resonators[1]. These have contributed to reducing narrow-band frequency noise travelling in a duct with fairly low-speed flow. The additional investigations have indicated that the resonance performance of a single resonator falls as flow speed is increased[2, 3]. The reason appears to be induced by entropy fluctuations in the shear layer [4]. Since such energy dissipations may occur close to the resonator entrance and are almost unable to be controlled, the multieffect of resonators is expected to result in the noise reduction level desired. Furthermore, a silencer which is composed of resonators with suitable configuration may be useful for reducing random frequency noise with less static pressure loss and secondary noise generation.

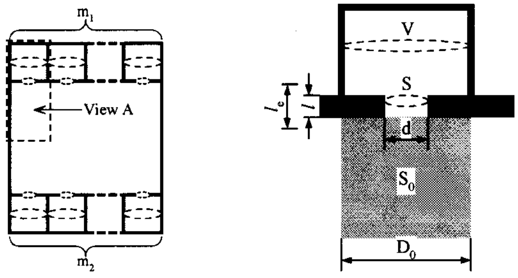
This study is aiming for a more efficient use of the multiple resonators for a duct with incompressible higher-speed flow. The silencer transmission loss has been calculated and measured, and its characteristics are examined in relation to Mach number of the flow, the arrangement of resonators, the number of resonators and their effective ranges of frequency.

2. *Equations*

A silencer composed of multiple Helmholtz resonators, which are arranged on either of the opposite sides of a rectangular duct terminating with the anechoic end, is depicted in Fig.1(a) (vertical section) and Fig.1(b) (cross section), where m_1, m_2 denote the number of transverse resonators, N the number of total resonators in the longitudinal direction, n_1, n_2,.......,n_i the number of longitudinal resonators having identical size, and L_1, L_2,.............,L_{N-1} connecting length of a duct between any two



(a) vertical section



(b) cross section (c) detail of view A

Fig.1 Model of silencers

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neighboring resonators. Fig.1(c) shows the detail of one resonator as shown in view A in Fig.1(b), where symbols are explained as follows ; V: volume of resonance chamber, l: actual length of connector, l_e: effective length of connector, d: diameter of connector, S: cross-sectional area of connector, and Do, So: width and cross-sectional area of a partial duct corresponding to individual resonator, respectively.

For an expansion chamber type silencer, Alfredson and Davies presented the theoretical treatment introducing the term of energy dissipation by entropy fluctuations into the momentum, energy, and continuity equations set up at a discontinuity [5]. On the basis of this concept, Munjal derived a transfer matrix in connection with sound pressure and mass rate at the duct sections directly at the front and back of a resonator [6]. If volume velocity is used instead of mass rate in the above matrix, matrix elements A,B,C,D may be rewritten as

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \frac{1}{2M + \frac{Z_r}{Z_0}} \begin{bmatrix} M + \frac{Z_r}{Z_0} & M^2 Z_0 \\ \frac{1}{Z_0} & M + \frac{Z_r}{Z_0} \end{bmatrix} \tag{1}$$

In eq. (1), M is Mach number of mean flow passing over the resonator and Z_o is the characteristic impedance of the duct given by

$$Z_0 = \frac{\rho_0 c}{S_0} \tag{2}$$

where ρ_o is mean density of medium, c is sound speed, and additionally Z_r is the acoustic impedance of a resonator given by

$$Z_r = R + j(X + R) \tag{3}$$

where R is connector resistance and X is resonator reactance with the resistance term omitted, which are shown in the ref. 7). Therefore the dimensionless impedance Z_r/Z_o may be expressed by

$$\frac{R}{Z_0} = \frac{16}{\left(\frac{d}{l_e}\right)\left(\frac{d}{D_0}\right)^2} \sqrt{\frac{\mu f}{\pi \rho_0 c^2}} \tag{4}$$

and

$$\frac{X}{Z_0} = S_0 \left(\frac{f}{f_r} - \frac{f_r}{f}\right) \sqrt{\frac{l_e}{VS}} \tag{5}$$

where μ denotes viscosity coefficient of medium, f frequency, and f_r resonance frequency given by

$$f_r = \frac{c}{2\pi} \sqrt{\frac{S}{Vl_e}}$$

The transfer matrix for a duct of connecting length L between two neighboring resonators can be obtained from the analysis in ref. 4), that is,

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = e^{-jkLM} \begin{bmatrix} \cos kL & jZ_0 \sin kL \\ \frac{j \sin kL}{Z_0} & \cos kL \end{bmatrix} \tag{6}$$

and

$$k = \frac{2\pi f}{c\left(1 - M^2\right)} \tag{7}$$

Thus sound pressure p_I and volume velocity q_I at the silencer entrance may be related to both quantities p_{II} and q_{II} at its exit, through a series of matrices which can be given as

$$\begin{pmatrix} p_1 \\ q_1 \end{pmatrix} = \begin{pmatrix} A_1 & B_1 \\ C_1 & D_1 \end{pmatrix} \begin{pmatrix} a_1 & b_1 \\ c_1 & d_1 \end{pmatrix} \begin{pmatrix} A_2 & B_2 \\ C_2 & D_2 \end{pmatrix} \begin{pmatrix} a_2 & b_2 \\ c_2 & d_2 \end{pmatrix} \cdots$$
$$\cdots \begin{pmatrix} a_{N-1} & b_{N-1} \\ c_{N-1} & d_{N-1} \end{pmatrix} \begin{pmatrix} A_N & B_N \\ C_N & D_N \end{pmatrix} \begin{pmatrix} p_{II} \\ q_{II} \end{pmatrix} \tag{8}$$

where subscripts of the matrix elements corresponds to the individual resonator longitudinally arranged and its rear duct which are numbered as shown in Fig.1(a). The above p_I, q_I, p_{II}, and q_{II} are written as

$$p_I = p_{Ii} + p_{Ir} \tag{9}$$

$$q_I = \frac{1}{Z_0}\left(p_{Ii} - p_{Ir}\right) \tag{10}$$

$$p_{II} = p_{IIi} \tag{11}$$

$$q_{II} = \frac{p_{IIi}}{Z_0} \tag{12}$$

where p_{Ii} and p_{Ir} are, respectively, the incident and reflected pressures at the silencer entrance and p_{IIi} transmitted pressure in the tail duct.

The transmission loss TL is defined as

$$TL = 10 \log \left| \frac{p_{Ii}}{p_{IIi}} \right|^2 \tag{13}$$

Each matrix for the resonator and partial duct may be obtained from eqs. (1)-(7), so that substituting eqs. (9)-(12) into eq. (8) and by removing p_{Ir}, one may get the acoustic energy ratio $|p_{Ii}/p_{IIi}|^2$. Then, using eq. (13), one could calculate the transmission loss for this resonance type silencer.

3. *Experimental Apparatus and Method*

The experimental apparatus used in this investigation is shown schematically in Fig.2. The sound was produced by an oscillator feeding through an amplifier and conducted to the system by means of two loud speakers. The sound which passed over the multiple Helmholtz resonators continued down through the tailpipe to the termination, which consisted of glass wool surrounded by an involute tube. The propagating signal was detected with the probe tube microphone traversing axially along the test

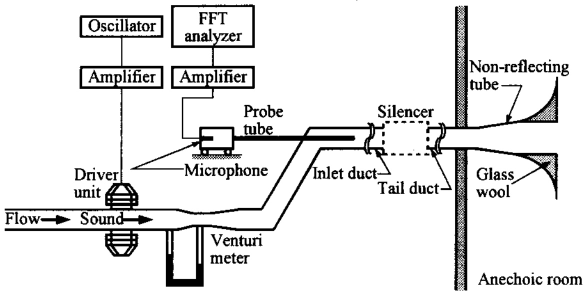

Fig.2 Schematic diagram for the experimental set-up.

section. The air flow from the blower whose noise was sufficiently reduced by a pre-muffler, progressing together with the sound from the driver unit, was emitted into the anechoic room. The mean flow velocity U was measured with a Venturi meter.

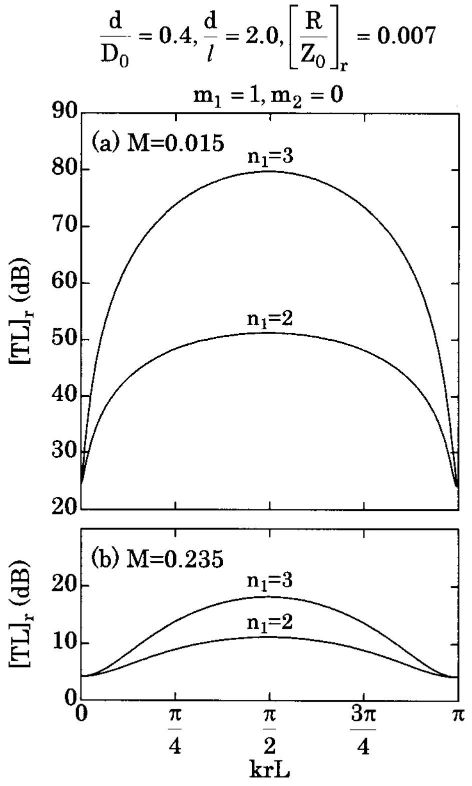

Fig.3 Relationship between the silencer transmission loss and phase angle at resonance frequency

The transmission loss at each frequency was obtained from the difference between the respective peak values of sound pressure level in the inlet duct and that in the tail duct. In this case, the additional sound pressure level brought about by the reflected wave from the connector discontinuity was corrected with the following equation.

$$NR = 10\log\left[\left|\frac{p_{Ii}}{p_{IIi}}\right| + \left|\frac{p_{Ir}}{p_{IIi}}\right|\right]^2 \qquad (14)$$

where NR denotes the measured sound pressure level difference as above-mentioned. Incidentally it has been assured a priori that the sound pressure propagated in the tail duct was unaffected by flow noise generated at the connector, and the termination had little sound reflected.

4. *Results*

4.1 *Silencers with the identical-size resonators*

The dimensionless parameters not including the number of resonators and Mach number will first be mentioned. The resistance/duct impedance ratio at the resonance frequency $[R/Z_o]_r$ is related to the maximum attenuation of the silencer given by the physical quantities at room temperature in eq. (4). The geometric parameter $\sqrt{VS/l_e}/S_o$ influences the resonator reactance and this in turn widens the frequency range in which the silencer will efficiently function, however, it is disassociated from the attenuation at the resonance frequency, as it vanishes at $f=f_r$ as seen in eq.(5). The resonance phase angle krL, determined by substituting f_r in terms of frequency f in eq.(6), is much

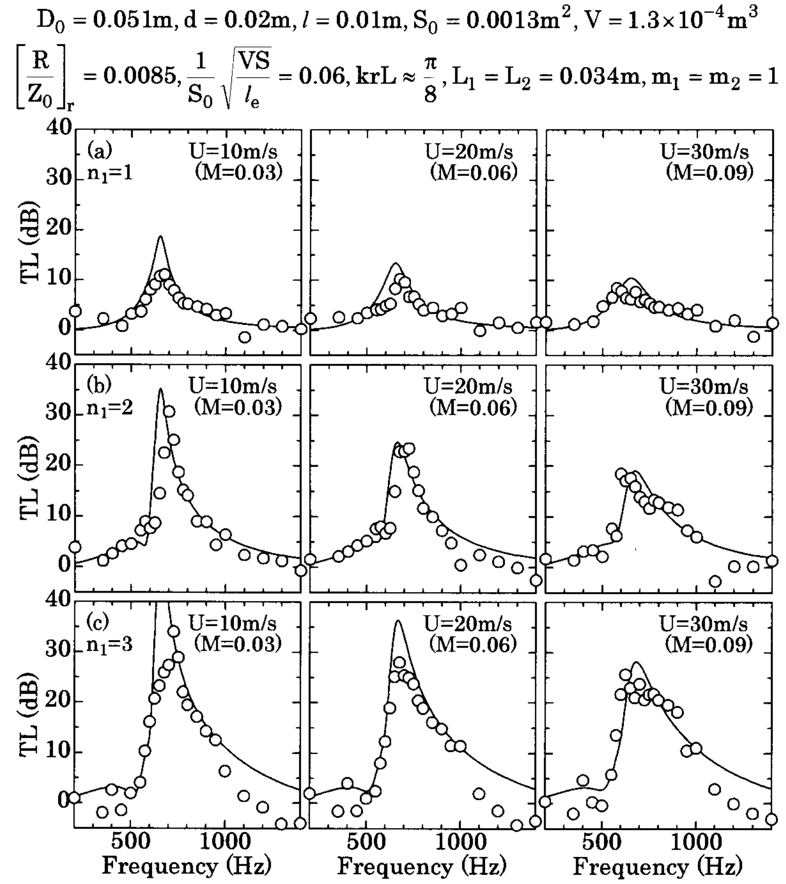

Fig.4 Transmission loss characteristics for the silencers with identical-size resonators

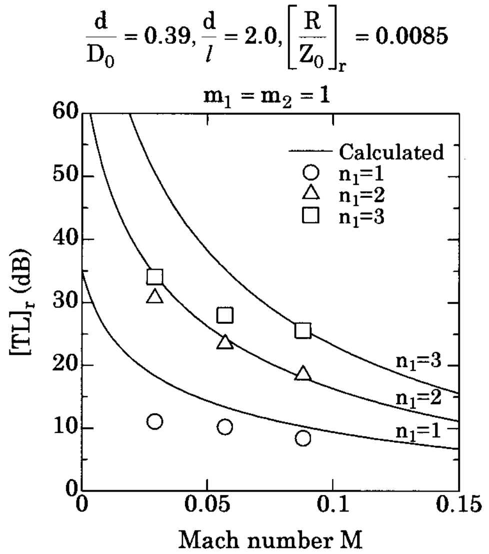

Fig.5 Effects of the number resonators(n_l) and Mach number on transmission loss at resonance frequency

associated with the silencer performance. Namely, as shown in Fig.3, when it is π/2, the maximum transmission loss expressed by $[TL]_r$ becomes largest, decreasing as its value approaches zero or π. In the case of this investigation, the above parameters are fixed in each presented datum. Additionally the effective length of the connector l_e is obtained, as reference to the former experimental results on the open-end correction for connectors[3].

The numerical and experimental results on the transmission loss characteristics for the silencers composed of the identical-size resonators functioning at mid-range frequency are shown in Fig.4. The calculated results indicate that the resonance effect diminishes with increase of the mean flow velocity U or Mach number M, but to the contrary as the number of longitudinal resonators n_l is increased. In this case, silencer transmission loss has non-symmetric variation in lower and higher regions with respect to the resonance frequency because the value of krL is about π/8, having gentle-gradient frequency characteristics with flow velocity. Experimental transmission loss fairly much agrees with those calculated, even though there are some cases where it is quite small as compared with the numerical results at the resonance frequency. This considerable agreement means that transverse resonators could be treated by equally portioning out plane wave front travelling in the duct with the resonator number, m_1+m_2, and their noise-reduction characteristics could be evaluated by using the partial cross-sectional area S_o, equivalent to individual resonator.

Figure 5 shows the numerical values of $[TL]_r$ along with the experimental data shown in the preceding figure. They vary with smaller gradients as the flow speed increases and will almost be unchangeable with the Mach number exceeding about 0.15. Thus the multieffect of resonators is markedly significant at considerably lower Mach number and, according to the calculations, may be kept to a certain extent up to the limit of the incompressible flow even though it decreases. In these data, for example, the maximums of experimental transmission loss for the multiple type when M =0.09, roughly speaking, are less by about 10 dB than that when M = 0.03, and have greater value by 15 dB at least at n_l=3 than that of a single resonator. Such resonance-frequency attenuations will be raised higher, if the value close to π/2 is taken as [krL]. The above-mentioned performance falls with the flow speed. This may be caused by the fact that the excess pressure oscillating the medium in the volume chamber is weakened by energy loss of flow passing over the resonators. That is hardly to be controlled by a method in fluid dynamics, however, the noise reduction level required of a silencer at narrow-band frequency could be obtained by the multieffect of a few or several Helmholtz resonators.

4.2 *Silencers with the different-size resonators*

The basic characteristics obtained numerically for the silencer composed of two different-size resonators mounted longitudinally are shown in Fig. 6. In these cases, while volume of one resonator is changed, that of the other remains fixed. When M = 0.015, transmission loss heightens at its bottom as the resonance frequency band width Δf_r becomes narrower. However, when M =0.235, the characteristic curve has the mountain-like shape forming a top at the mid-frequency range in the bottom region referred above and is similar to a part of characteristic curve for the fundamental expansion chamber type silencer which cycles every phase angle, that is, every π. The fact implies that the double effects contributed by both resonators predominates over the resonance components which are originally weakened with the higher-speed flow.

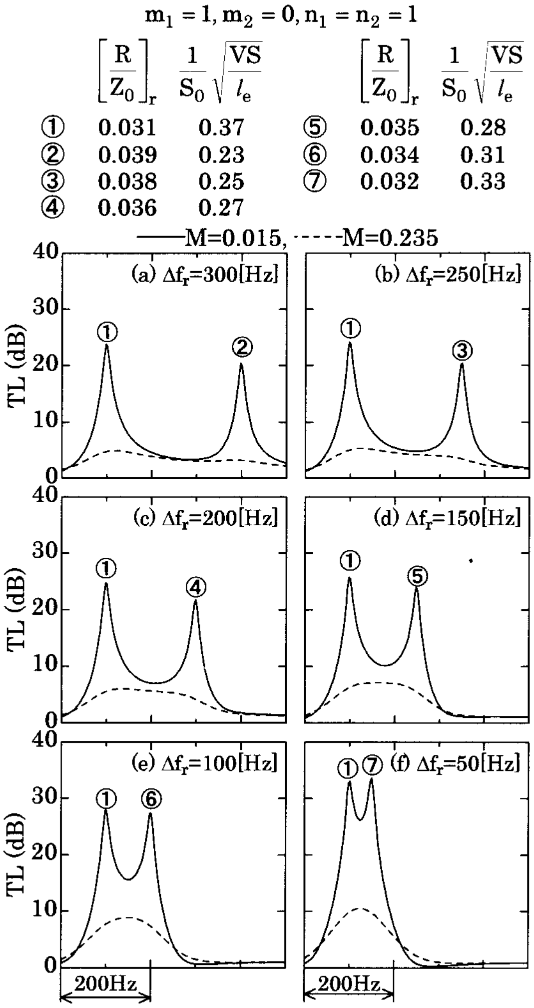

Fig.6 Changes of transmission loss level and frequency range for the silencers with two different-size resonators

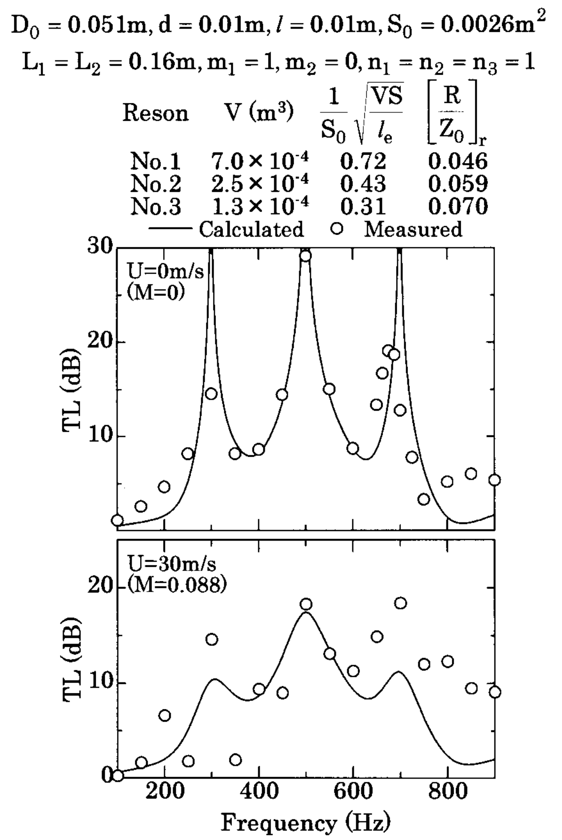

Fig.7 Transmission loss characteristics for the silencer with three different-size resonators

Figure 7 shows transmission loss characteristics for three resonators, that is, $n_1= n_2 = n_3= 1$ and $m_1= 1$. The experimental data for M = 0.09 apparently give a variation on the whole along the calculated curve which has the three ridges covering the bottom level over the range at least about twice as wide as Δf_r, and this is in accordance with the case without flow. Especially the top attenuation can be raised higher because it is doubled by the resonance effect in the middle ridge. As Δf_r diminishes, the curve of the transmission loss characteristics will tend towards the mountain-like shape and the top of the curve would have a higher peak than that of the basic combinations shown in Fig. 6. Additionally it is natural that if the number of the longitudinal resonators is further increased, the predominant component will also increase one by one, and the frequency range of attenuation would be widen out.

Such silencers have the following excellent properties compared with the expansion chamber type as can be summarized here.

(a) It is comparatively easy to adjust the predominant components of attenuation to the main frequency components of noise which should be reduced, since these silencers have no cyclic characteristic.

(b) Excluding the application to particularly small-sized piping systems, the space necessary for an equipment could be smaller than a duct, since the level and frequency of the predominant components are mainly influenced by the resonance chamber volume.

(c) The flow noise secondarily generated and static pressure loss may be much lightened.

Thus the silencer with the different-size Helmholtz type resonators may be applicable to controlling of random noise at low and mid-range frequency where absorbing material may not be so much useful.

5. *Conclusions*

Calculations and measurements have been made of transmission loss for the Helmholtz type resonators in a duct. The results are summarized as follows:

(1) The numerical and experimental characteristics are in fairly good agreement as a whole.

(2) The transverse arrangement of resonators can be treated by equally portioning out the plane wave front with the partial cross-sectional area equivalent to individual resonator.

(3) The longitudinal arrangement has the multifunction of increasing the resonance effect, even though the performance of each resonator may be lowered by the shear layer close to its entrance.

(4) In the above case, the identical-size resonators bring about the rises of the noise reduction level in the narrow frequency range according to the given flow speed, and the different-size resonators of three kinds or more could reduce the random frequency noise accompanying a flow at higher speed.

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