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Secondary Compression Behavior in Standard Consolidation Tests

by

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Abstract

One-dimensional consolidation analysis is described for predicting the consolidation time curve of clays exhibiting secondary compression during primary consolidation. The constitutive soil model is based on the equation governing the secondary compression rate of the decrease in void ratio. This model uses four parameters, namely, C_c, C_c^*, C_α and c_v^*, that can be easily determined or assumed from conventional standard oedometer test results. To check the validity of the proposed soil model, the consolidation time curves observed in oedometer specimens are compared with those predicted by the analysis. A satisfactory agreement can be obtained between the computed behavior and oedometer observations. In addition, it is shown that the void ratio rate due to secondary compression during primary consolidation varies by approximately two times its final value before the application of the next loading increment.

Keywords: *One-dimensional consolidation, Secondary compression, Void ratio rate, Explicit finite analysis*

1. *Introduction*

Currently, Terzaghi's consolidation theory and the result of the standard one-dimensional consolidation test are used to predict the consolidation settlement-time curve in a field. From the experimental data and the field observation, it is well known that the actual rate of consolidation differs from the rate predicted by Terzaghi's theory. This difference between the prediction and the observation is attributed to secondary compression effects. The most notable secondary compression is proportional to the logarithm of time and can be observed after the completion of primary consolidation based on Terzaghi's theory in which a linear relationship between void ratio and effective stress is assumed.

Taylor and Merchant (1940) were the first to present a secondary compression model and since then secondary compression has been a key to solving the consolidation problem[1]. Taylor (1948) and many other researchers stated that secondary compression occurs during primary consolidation as well as after the end of primary consolidation. The literature on secondary compression is extensive and there is a brief review by Imai (1995)[2]. In most studies, one-dimensional consolidation is assumed to involve two process, namely, primary and secondary consolidations and also both processes are assumed to begin simultaneously with consolidation. However, it is very difficult to confirm secondary compression behavior during primary consolidation because the total compression, namely, the sum of primary and secondary compressions only can be observed in the conventional consolidation test and the compressions are inseparable. Nevertheless, most studies (Wahls, 1962[3]; Mesri, 1974[4]; Murakami, 1980[5]; Akaishi, 1980[6] among others) have focused on the characteristics of secondary compression during the stage of primary consolidation.

The main area of debate (Ladd, 1976[7]; Mesri, 1985[8]; Leroueil, 1988[9]) has been whether creep is significant during primary consolidation. Leroueil et al. (1985) [10] pointed out that the constitutive equations in the form of $F(e,\sigma',t)=0$ encounter a major difficulty when an origin for time must be defined although secondary compression decreases logarithmically with time. A unique constitutive equation in the form of $F(e,\sigma',\partial e/\partial t)=0$ was used to avoid this difficulty (Sekiguchi, 1976[11]; Leroueil, 1985[10]; Imai, 1992[12]). However, none of the existing constitutive equations of clays for one-dimensional consolidation provide a clear explanation of secondary compression behavior during primary consolidation.

It is the purpose of this study to clarify secondary compression behaviors during primary consolidation, to provide one-dimensional consolidation analysis for constitutive equation in the form of $F(e,\sigma',\partial e/\partial t)=0$ and to compare the solution with oedometer test results.

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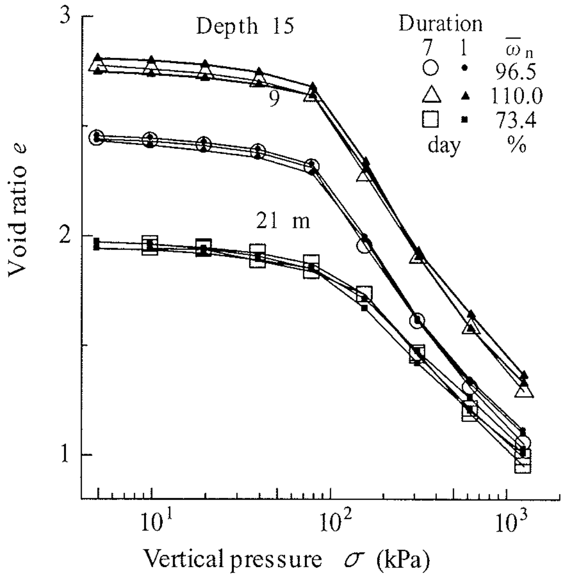

Fig. 1 Observed $e - \log\sigma$ curves for different increment durations

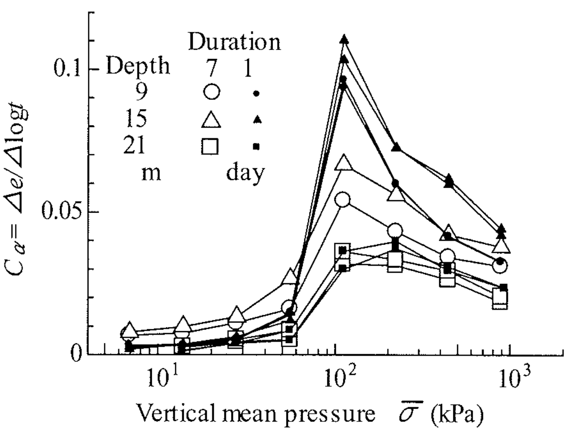

Fig. 2 Coefficient of secondary compression C_α vs vertical effective stress

2. *Oedometer tests and clays tested*

One-dimensional consolidation tests were performed on undisturbed clays. The undisturbed samples carefully trimmed to fit the consolidometer ring were 6 cm in diameter and initially 2 cm in height. The rings were lubricated with silicon grease to reduce side friction, and filter papers were placed on the top and bottom of the sample. In all cases, drainage was permitted at both ends of the sample. A small seating stress, usually 9.8kPa, was applied, and after 60 min, the sample was submerged. The constant-water-temperature bath was maintained at 15±2°C.

The pressure increment ratio, $d\sigma / \sigma = 1$, and two increment durations, 1 and 7 days, were used to study the secondary compression characteristics. In this study, σ is the vertical effective stress and the prime for the effective stress is omitted.

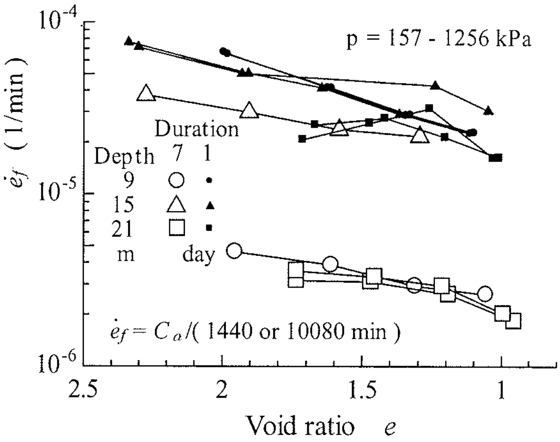

Fig. 3 Void ratio vs void ratio rate at end of consolidation

All undisturbed Hitachi clays were obtained 8～23m below the ground surface using a thin-walled, fixed-piston sampler. The natural water content varied from approximately 82% to 121%. The liquid limit range is 96% to 131% and the plastic limit range is 44% to 54%.

3. *Laboratory investigation*

Figure 1 shows plots of the void ratio e versus the logarithm of vertical effective stress $\log\sigma$ relationships for clay samples taken from depths of 9 to 21 m. Two identical clay samples were subjected to two different increment durations for consolidation. One sample was loaded for one day at each pressure increment, whereas the consolidation duration of the other sample was 7 days. The most probable value of the preconsolidation pressure p_c for all tests is approximately 80 kPa.

It will be seen from the legends accompanying Fig. 1 that there is no appreciable change in $e - \log\sigma$ relationships with the different durations of pressure increment. It is clear that $e - \log\sigma$ relationships are independent of these consolidation durations. The result in Fig. 1 is completely different from the corresponding consolidation test result in Crawford's paper (1964)[13].

In Fig. 2, the coefficient of secondary compression, C_α, is plotted against the vertical effective pressure. The magnitude of C_α increases rapidly to a maximum for pressures higher than the preconsolidation pressure and then decreases. As the pressure increment duration increases, the magnitude of C_α, which is indicated with a large symbol in Fig. 2, decreases. The void ratio rate at the end of consolidation can be calculated using C_α and consolidation duration.

In Fig. 3, $\dot{e}_f$ in the range of normal consolidation is plotted against void ratio at the end of consolidation. The magnitude of $\dot{e}_f$ varies with void ratio and consolidation duration. It has been shown in numerous studies that there is a unique $e-\log\sigma$ relationship in normally consolidated clays.

From the author's oedometer test results, however, it is difficult to observe that the relationship of $e-\log\sigma$ of normally consolidated clays shows a constant $\dot{e}_f$.

4. *Development of theory*

The one-dimensional consolidation process is assumed to involve two components, namely, primary- and secondary-consolidation rates. The void ratio rate $\dot{e}$ is the sum of primary and secondary consolidations. Moreover, $\dot{e}$ values due to both consolidations are assumed to begin simultaneously with the application of pressure. The new secondary compression has a completely different behavior from the secondary consolidation before consolidation. The equation that governs the consolidation of saturated clay undergoing one-dimensional compression and drainage can be expressed as

$$\frac{\partial e}{\partial t}=(\dot{e}_p+\dot{e}_s)=(1+e_0)\frac{\partial}{\partial y}\frac{k}{\gamma_\omega}\frac{\partial u}{\partial y} \quad (1)$$

in which $\dot{e}_p$ and $\dot{e}_s$ are the void ratio rates of primary and secondary consolidations, respectively, e_0 is the initial void ratio, k is the permeability, γ_ω is the unit weight of water, t is the elapsed time, y is the vertical coordinate and u is the excess pore water pressure. During one-dimensional consolidation, the void ratio e at any time t can be calculated from

$$e=e_0-C_c^*\log(\sigma/\sigma_0)+C_\alpha\log(\dot{e}_s/\dot{e}_i) \quad (2a)$$

$$\dot{e}_p=-\frac{0.434C_c^*}{\sigma}\dot{\sigma}=-m_p\dot{\sigma} \quad (2b)$$

$$\dot{e}_s=\dot{e}_i\times10^{-\chi/C_\alpha} \quad (2c)$$

$$\chi=e_0-C_c^*\log(\sigma/\sigma_0)-e \quad (2d)$$

in which C_c^* is the compression index defined by the primary compression, σ_0 is the initial effective stress, χ is the secondary compression and $\dot{e}_i$ is the initial void ratio rate for the secondary compression at $\chi=0$. Substituting Eq. (2b) and Eq. (2c) into Eq. (1) then gives

$$\frac{\partial u}{\partial t}=c_v^*\frac{\partial^2 u}{\partial y^2}-\dot{e}_s/m_p \quad (3)$$

in which the coefficient of consolidation $c_v^*=k(1+e_0)/\gamma_\omega/m_p$.

Equation (3) can be solved by the explicit finite difference method. If written infinite difference form, Eq. (3) becomes

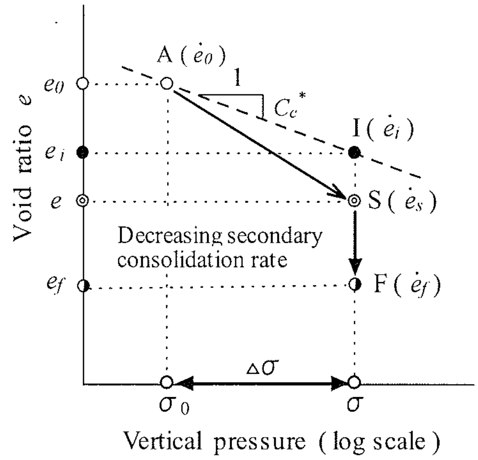

Fig. 4 Schematic diagram for $e-\log\sigma$ relations

$$u_{y,t+\Delta t}=u_{y,t}+M(u_{y-\Delta y,t}-2u_{y,t}+u_{y+\Delta y,t})-\dot{e}_s\times\Delta t/m_p \quad (4)$$

in which $M=c_v^*\Delta t/(\Delta y)^2\leqq 1/2$.

To describe the one-dimensional compression of clays exhibiting secondary compression, Figure 4 shows a unique relationship between void ratio, effective stress and void ratio rate.

In Fig. 4, point A shows the initial effective stress σ_0 and the initial void ratio e_0 before the application of pressure. The compression index C_c^* is independent of secondary compression and is defined by the slope of the AI line in Fig. 4. The difference in void ratio e_0-e_i , immediately after the increase in effective stress, is related to primary consolidation. The void ratio rate at point I is denoted by $\dot{e}_i$. Secondary consolidation continues at a decreasing rate from I towards S and F. The void ratio rate at S due to secondary consolidation is expressed by Eq. (2c). The difference in void ratio e_i-e_0 leads to secondary consolidation at any time t. In the standard one-dimensional consolidation test, the consolidation duration is equal to 1440 minutes and its total compression is used to determine $e-\log\sigma$ relations and C_c.

If the conventional compression index C_c is obtained from the void ratio change e_0-e_f in Fig. 4, $\dot{e}_i$ can be calculated by

$$\dot{e}_i=\dot{e}_f\times10^{(C_c-C_c^*)\log(\sigma/\sigma_0')/C_\alpha} \quad (5)$$

in which $\dot{e}_f$ is the void ratio rate at point F.

4.1 *Example of calculation*

To illustrate the one-dimensional consolidation behaviors of the proposed soil model, it has been used to calculate the void ratio and the void ratio rate-time

Table 1 Soil parameters

C_c	C_c^*	c_v^*	C_α	e_0 a)	$\dot{e}_f$ b)
1.5	1~1.5	0.1	0.05	3	1.5×10^{-5}

c_v^* (cm²/min), a) e_0 at σ_0 = 39.2kPa, b) elapsed time=1440min (1/min)

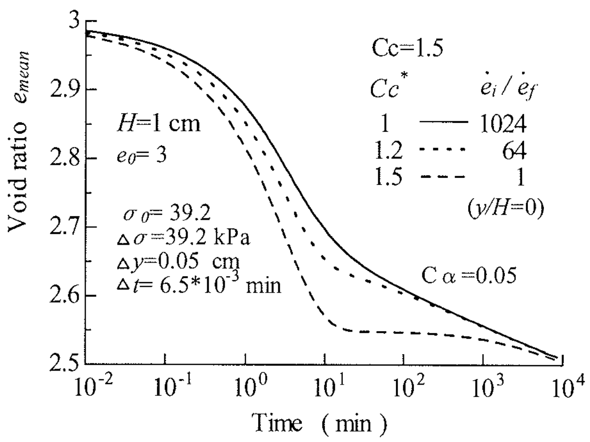

Fig. 5 Average void ratio-time curves

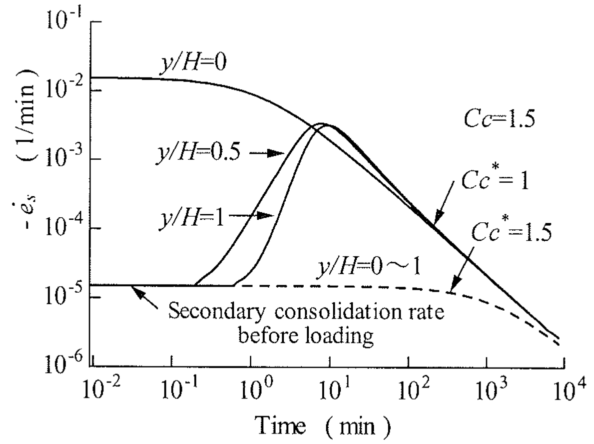

Fig. 6 Void ratio rate-time curves

curves for typical oedometer specimens. For a saturated clay sample loaded instantly under one-dimensional compression and drainage, the initial and the boundary conditions are

$$\left.\begin{array}{l} u(y=0\ ,\ t>0)=0 \\ u(H\geqq y\geqq 0,\ t=0)=u_0(=\Delta\sigma) \\ \partial u/\partial y(y=H\ ,\ t>0)=0 \\ \dot{e}_0(=\dot{e}_i)(H\geqq y\geqq 0\ ,\ t\leqq 0)=\dot{e}_f \end{array}\right\} \quad (6)$$

in which H is the length of the longest drainage path, u_0 is the initial excess pore water pressure, and the void ratio rate immediately before the application of loading $\dot{e}_0(=\dot{e}_i)$ is assumed to be equal to one at the end of consolidation using Eq. (7). The material constants used in the calculations are listed in Table 1.

$$\dot{e}_f = 0.434 C_\alpha /(elasped\ time) \quad (7)$$

Examples of numerical analysis of Eq. (4) for the special cases in which three types of C_c^* and $C_c(=1.5)$ are maintained constant are shown in Fig. 5. The ratio of the compression indexes C_c^*/C_c is the ratio of primary consolidation to total consolidation, which is the sum of primary and secondary consolidations. The shape of consolidation-time curves depends on C_c^*/C_c. If this ratio is equal to 1, only secondary compression before loading occurs after the application of a load. As can be seen from the broken line in Fig. 5, therefore, the secondary compression is small during primary consolidation and the consolidation -time curve approximately corresponds to Terzaghi's rigorous solution. It can also be seen from the dotted line and the solid line in Fig. 5 that the secondary compression during primary consolidation increases as C_c^*/C_c decreases and the magnitude of secondary compression rate after the completion of primary consolidation becomes to $C_\alpha=0.05$ used in this calculation. From these calculations, the rates of secondary compression, $\dot{e}_s$, inside the consolidation layer during one-dimensional consolidation are shown in Fig. 6. It is important to note that the broken lines in Fig. 5 and Fig. 6 are calculated using $C_c^*/C_c=1$ and the magnitude of secondary compression rate is approximately constant during primary consolidation. Solid lines in Fig. 6 show the typical change in the rate of secondary compression immediately after the beginning of consolidation. It is important to realize that the secondary consolidation rate increases with consolidation time and tends to converge at the same value at the later stage of consolidation. During primary consolidation, secondary compression expressed by the proposed model is not proportional to the logarithm of time.

4.2 *Comparisons of calculated and experimental result*

To compare an experimental consolidation-time curve with calculated ones, the values of C_c, C_c^*, C_α and c_v^* must be known. The compression index C_c and the coefficient of secondary compression C_α are easily determined from $e-\log\sigma$ relationships and the final slope of the semi-logarithm plot in conventional oedometer test results, respectively. The coefficient of consolidation c_v, which is determined from the graphical solution for consolidation time curves, such as the square root of the time method, is highly influenced by secondary compression. It is very difficult to determine the values of c_v^* and C_c^*, which are independent of secondary compression effects.

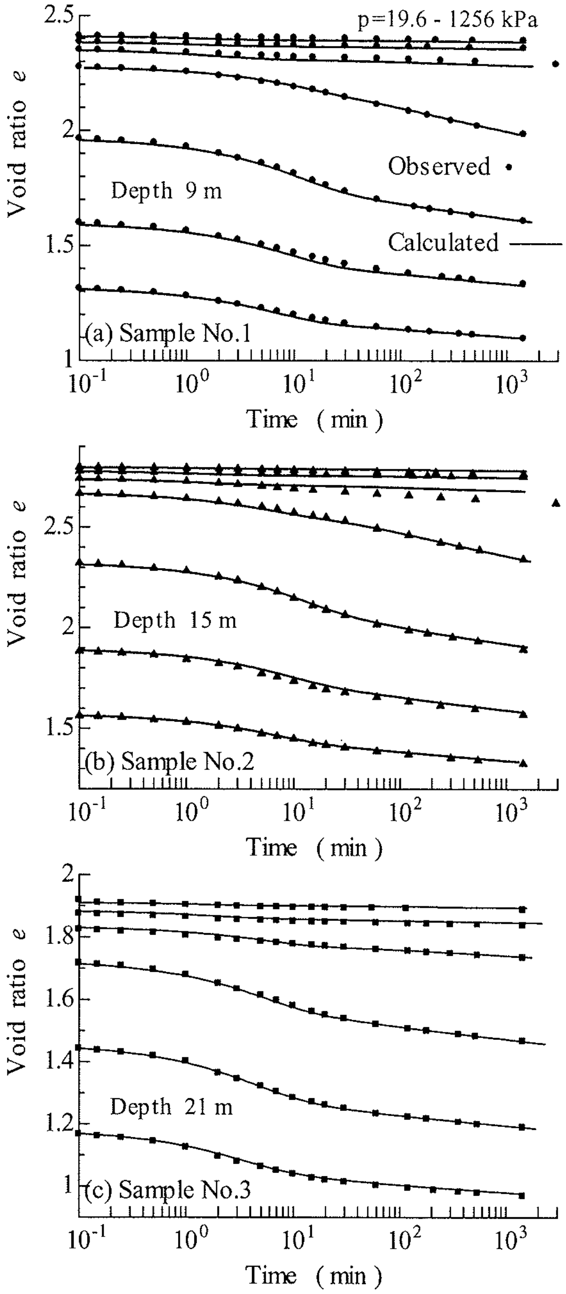

Fig. 7 Void ratio rate-time curves for the one-day loading duration

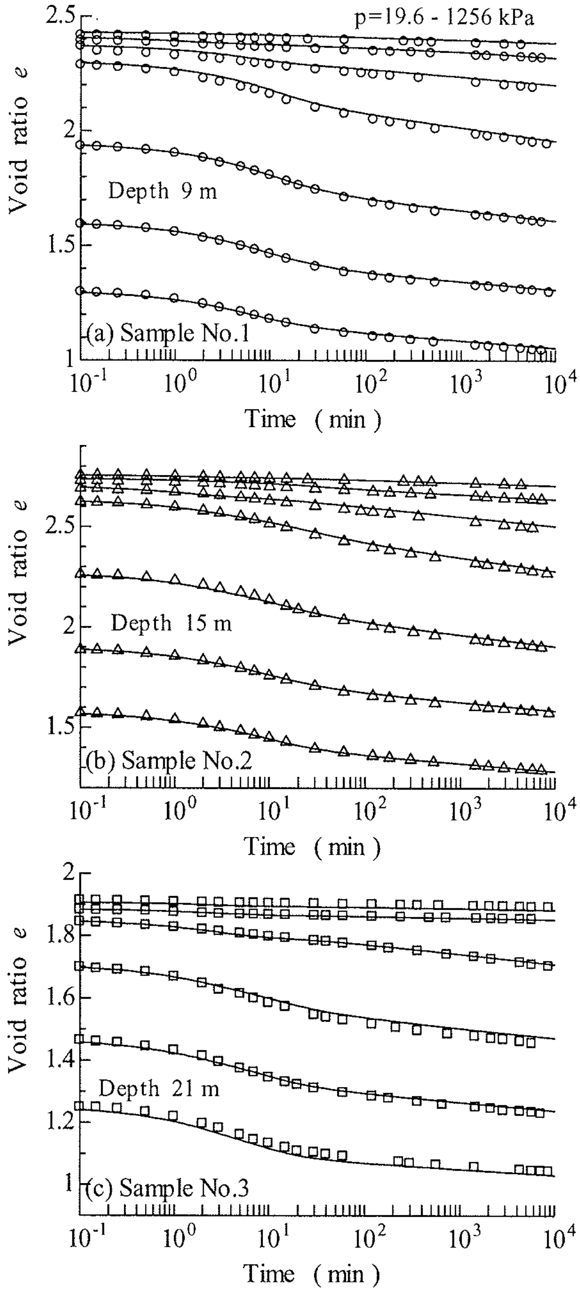

Fig. 8 Void ratio rate-time curves for the one-week loading duration

In the present treatment, the following simplified approach was adopted. The curve ruler method adopted by the Japanese Geotechnical Society was used to determine the approximate values of c_v^* and C_c^*. To reduce secondary compression effects on c_v^*, the relation of the average degree of consolidation, U, and the time factor, T_v, up to $U=50\%$ is fitted to the observed data. The first approximate value is obtained from the time t_{50} for $U=50\%$ and the value of C_c^* is calculated using the consolidation settlement d_{50}, which corresponds to t_{50}. Using approximate values for soil indexes, the results of preliminary calculation for the consolidation-time curve were compared with the observed ones and the values of c_v^* and C_c^* were modified by a trial and error method to obtain the closeness of the calculated and observed curves with the aid of a curve fitting procedure. Figures 7 and 8 show the calculated void ratio-time curves for the three specimens together with the observed values that were performed to two kinds of loading durations.

The closeness of the calculated and observed curves is an indication of the reliability of the proposed model and the success of the above curve fitting method.

Figure 9 shows the ratio of the compression index C_c^* / C_c used in the calculation. This means the ratio of primary consolidation to the total one at the end of con-

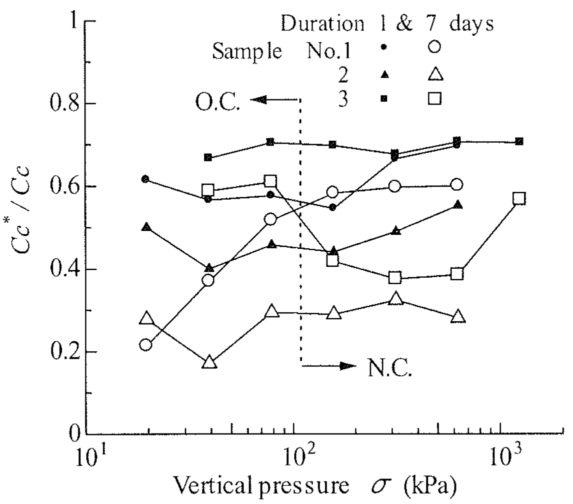

Fig. 9 Relationship between C_c^* / C_c and vertical pressure

solidation immediately before the application of the next step loading increment. It can be seen from Fig. 9 that the ratio of compression index is independent of the magnitude of consolidation pressure and increases as the consolidation duration decreases.

5. *Conclusions*

The main conclusions obtained from the experimental study and the numerical analysis are summarized as follows,

(1) Relationship between the void ratio and the logarithm of vertical effective stress is independent of load duration. The void ratio rate $\dot{e}_f$ of normally consolidated Hitachi clays at the end of consolidation varies with the decrease in the void ratio. It can be said that there is not always a unique relationship between $e-\sigma-\dot{e}$.

(2) The void ratio rate $\dot{e}_s$ for secondary compression during primary and secondary consolidations may be expressed as a function of the magnitude of secondary compression generated during consolidation. Five parameters are required for this proposed model. They are the compression indexes C_c and C_c^*, the coefficient of consolidation c_v^*, the coefficient of secondary compression C_α and another parameter $\dot{e}_i$. Each parameter can be easily determined from the results of a standard consolidation test and by trial and error.

(3) The method of analyzing the one-dimensional consolidation of clays exhibiting secondary compression appears to give reliable predictions of the consolidation-time curves for laboratory consolidation tests.

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