

# Vibroacoustic Coupling of a Cylindrical Enclosure with an Excited End Plate

by

Atsushi KOJIMA<sup>\*1</sup>, Hiroyuki MORIYAMA<sup>\*2</sup> and Yasuo OSHINOYA<sup>\*2</sup>

(Received on Apr. 14, 2011 and accepted on Jun. 08, 2011)

## Abstract

Being extremely minute in comparison with other sources of industrial energy, acoustic energy must be amplified for use in industrial applications. In the present study, we consider a cylindrical structure with thin plates at both ends and investigate vibroacoustic coupling between the plate vibrations and the internal sound field when an external harmonic force is applied to one end plate, regarding such a coupling phenomenon as one of the available measures to amplify acoustic energy. This coupling is investigated theoretically and experimentally by considering the behavior of both plates and the acoustic characteristics of the internal sound field for various dimensions of the end plates and the cylinder. In the analytical model, the end plates are supported circumferentially by springs so that the support conditions can be made to closely match the actual conditions in the experiment by adjusting the stiffnesses of the springs. Then, in order to simplify this problem, the cylinder is assumed to be structurally and acoustically rigid at the lateral wall between the structure and the sound field. The dimensions and phase differences between plate vibrations, which maximize the sound pressure level inside the cavity, are clarified theoretically. These theoretical results are verified experimentally through an excitation experiment using an experimental apparatus that is emulated by the analytical model. The above-described results reveal that vibroacoustic coupling phenomena are useful for the amplification of the acoustic energy.

**Keywords:** Vibroacoustic Coupling, Cylindrical Enclosure, Structural Vibration, Internal Sound Field

## 1. Introduction

Although we are constantly surrounded by vibrations and sounds, most of these dissipate into the environment and are not used as sources of industrial energy. In particular, acoustic energy is extremely minute in comparison with vibration energy, and so applications of acoustic energy in mechanical engineering have been limited. However, in order to promote energy savings and suppress the greenhouse effect, the application of acoustic energy, e.g., sound-generated electricity from a voice obtained via piezoelectric elements installed on elastic plates, is regarded as advantageous. Although this electric generation system was developed as a supplementary electricity source for a mobile phone<sup>1)</sup>, this system is being investigated for application in other devices<sup>2)</sup>. Since the electricity is generated by both sound and vibration systems, vibroacoustic coupling between both systems is one means of

amplifying the acoustic energy. However vibroacoustic coupling is not used in the above-mentioned example.

In order to develop a new electricity generation system, we adopt an analytical model similar to the above-mentioned cylindrical structure with plates at both ends, because the vibration area of the model, on which piezoelectric elements can be installed, is twice as large as that in the case of a single plate. In almost all related studies, vibroacoustic coupling was estimated by assuming that the plate and cavity dimensions and the phase difference between the vibrations of the two plates were fixed. However, the eigenfrequencies of the plate and cavity vary with the dimensions of the plate and cavity, and the phase difference directly affects the sound field when the medium is much less dense than the plate. Accordingly, in this investigation, the dimensions of both the plate and the cylinder are varied over a wide range while a harmonic point force is applied to only one end plate. Vibroacoustic coupling that occurs between plate vibrations and the sound field in the cavity is theoretically and

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\*1 Graduate Student, Course of Science and Technology

\*2 Professor, Department of Prime Mover Engineering

experimentally investigated in terms of the vibration and acoustic characteristics. In particular, the phase difference between the vibrations of the two plates is considered to be a significant characteristic of the vibrations, and the conditions for maximizing the sound pressure level in the cavity is clarified in order to apply the coupling phenomena to the new generation system.

## 2. Analytical method

### 2.1 Analytical model

The analytical model considered herein consists of a cavity with two circular end plates, as shown in Fig. 1. The plates are supported by translational and rotational springs distributed at constant intervals and the support conditions are determined by their respective spring stiffnesses,  $T_1$  and  $T_2$ , and  $R_1$  and  $R_2$ , where the suffixes 1 and 2 indicate plates 1 and 2, respectively. The dimensions (i.e., the radius  $a$  and the thickness  $h$ ) of the plates, which have Young's modulus  $E$  and Poisson's ratio  $\nu$ , are varied. The sound field is assumed to be cylindrical and to have the same radius as the plates, and the length of the sound field is assumed to vary with the cylinder length. The boundary conditions are considered to be structurally and acoustically rigid at the lateral wall between the structure and the sound field. The coordinates used are the radius  $r$ , the angle  $\theta$  between the planes of the plates and the cross-sectional plane of the cavity, and the distance  $z$  along the cylinder axis. The harmonic point force  $F_1$  is applied to plate 1 at  $r_1$  divided by  $a$ , and  $\theta_1$  is fixed at 0 deg. The natural frequency of the plate is used as its excitation frequency.

### 2.2 Coupling equipment between plates and cavity

In order to formulate the plate motion, Hamilton's principle based on the variational principle is applied to the analytical model<sup>3)</sup> and the flexural displacements  $w_1$  and  $w_2$  on plates 1 and 2 are expressed by substituting Eq. (2) for the plate mode shape into Eq. (1) and are expanded over two sets of suitable trial functions:

$$\left. \begin{aligned} w_1 &= \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{1nm}^s X_{nm}^s e^{j(\omega t + \phi_1)}, \\ w_2 &= \sum_{s=0}^1 \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} B_{2nm}^s X_{nm}^s e^{j(\omega t + \phi_2)}, \\ X_{nm}^s &= \sin(n\theta + s\pi/2)(r/a)^m, \end{aligned} \right\} \quad (1)$$

$$(2)$$

where  $n$ ,  $m$ , and  $s$  are, respectively, the circumferential order, the radial order, and the symmetry index for the  $z$ -axis of the vibration mode with respect to the plate vibration. In addition,  $B_{1nm}^s$  and  $B_{2nm}^s$  are the coefficients to be determined,  $\omega$  is the angular frequency of the harmonic point force on the plate,  $t$  is the elapsed time, and  $\phi_1$  and  $\phi_2$  are the phases of the

respective plate vibrations. In this analysis,  $\phi_1$  is set to 0 deg, and  $\phi_2$  varies in the range of 0 to 180 deg. The equations of plate motion are obtained by finding the extremum of Hmliton's function in terms of Eqs. (1).

The acoustic modal shape  $Y_{npq}^s$  and angular resonance frequency  $\omega_{npq}$  in the cavity (where the indexes  $n$ ,  $p$ , and  $q$  indicate the circumferential, radial, and longitudinal orders, respectively) are defined as

$$Y_{npq}^s = \sin(n\theta + s\pi/2) J_n(\lambda_{np} r) \cos\{(q\pi/L)z\}, \quad (3)$$

$$\omega_{npq} = c \left\{ \lambda_{np}^2 + (q\pi/L)^2 \right\}^{1/2}, \quad (4)$$

where  $J_n$  is the  $n$ th-order Bessel function, and  $\lambda_{np}$  is the  $p$ -th solution of an eigenvalue function for a circular sound field having  $(n,p)$  modes divided by the radius. The boundary conditions between the plate vibrations and the sound field on the respective plate surfaces are found by assuming continuity of velocities on the plates:

$$\left( \frac{\partial P_c}{\partial \mathbf{u}} \right)_{z=0} = \rho_c \omega^2 w_1, \quad \left( \frac{\partial P_c}{\partial \mathbf{u}} \right)_{z=L} = -\rho_c \omega^2 w_2, \quad (5)$$

where  $P_c$  and  $\rho_c$  are, respectively, the sound pressure on each boundary surface (e.i., each plate surface) and the fluid density in the cavity, and  $\mathbf{u}$  is the unit normal to the corresponding plate surfaces, such that  $\partial P_c / \partial \mathbf{u} = 0$  on the lateral wall of the cylinder. Moreover, in the present analysis we assume that there is no sound source, and hence by using the above boundary conditions and Green's function for a non-uniform cavity with non-rigid walls,  $P_c$  can be expanded in terms of the acoustic modes consisting of a trio of orders,  $n$ ,  $p$  and  $q$ , as follows:

$$P_c = -\int_{A_1} G \rho_c \omega^2 w_1 dA_1 + \int_{A_2} G \rho_c \omega^2 w_2 dA_2 = \rho_c c^2 \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{P_{npq}^s Y_{npq}^s}{M_{npq}^s} \quad (6)$$

where  $G$  is Green's function,  $A$  is the total surface area of the plates, and  $P_{npq}^s$  and  $M_{npq}^s$  are, respectively, the pressure coefficients to be determined and the modal generalized mass of the  $n$ th acoustic modes.

When one end plate excited by point force is coupled by a cavity, the coupling equations between the plate vibrations and the sound field are expressed as<sup>4,5)</sup>

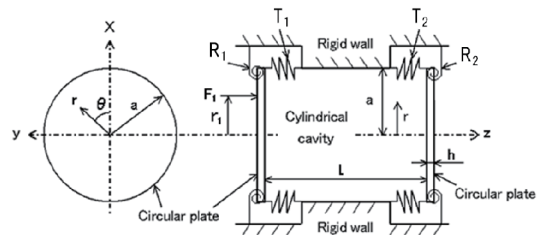


Fig. 1 Configuration of the analytical model

$$\left[ \sum_{m'=0}^{\infty} \left\{ K_{1nm}^s (1 + j\eta_p) - \omega^2 M_{1nm}^s \right\} + \sum_{m'=0}^{\infty} a F_{sn} \left\{ T_1 + \left( \frac{m}{a} \right) \left( \frac{m'}{a} \right) R_1 \right\} \right] \mathbf{B}_{1nm}^s e^{j\phi_1}$$

$$= \mathbf{F}_{1nm}^s + \frac{\rho_c c^2 \omega^2 A^2}{V_c} \sum_{m'=0}^{\infty} \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{I_1 (I_1 B_{1nm}^s e^{j\phi_1} - I_2 B_{2nm}^s e^{j\phi_2})}{M_{npq}^s (\omega_{npq}^2 + j\eta_c \omega_{npq} \omega - \omega^2)}, \quad (7)$$

$$\left[ \sum_{m'=0}^{\infty} \left\{ K_{2nm}^s (1 + j\eta_p) - \omega^2 M_{2nm}^s \right\} + \sum_{m'=0}^{\infty} a F_{sn} \left\{ T_2 + \left( \frac{m}{a} \right) \left( \frac{m'}{a} \right) R_2 \right\} \right] \mathbf{B}_{2nm}^s e^{j\phi_2}$$

$$= -\frac{\rho_c c^2 \omega^2 A^2}{V_c} \sum_{m'=0}^{\infty} \sum_{p=1}^{\infty} \sum_{q=0}^{\infty} \frac{I_2 (I_1 B_{1nm}^s e^{j\phi_1} - I_2 B_{2nm}^s e^{j\phi_2})}{M_{npq}^s (\omega_{npq}^2 + j\eta_c \omega_{npq} \omega - \omega^2)}, \quad (8)$$

where  $K_{1nm}^s$  and  $K_{2nm}^s$ , and  $M_{1nm}^s$  and  $M_{2nm}^s$  are components of the symmetrical stiffness and mass matrices, respectively, because the index  $m'$  is the radial order ( $m = m'$ ). In addition,  $\eta_p$  is the structural damping factor of the plate, and  $F_{sn}$  is a coefficient that is determined by indices  $n$  and  $s$ <sup>(3)</sup>. On the right-hand side of Eq. (7), the first and second terms express the point force and coupling between each plate vibration and the sound field, respectively, whereas the right-hand side of Eq. (8) contains only the coupling term. Here,  $c$  is the speed of sound in the cavity, and  $V_c$  is the volume occupied by the cavity. Moreover,  $I_1$  and  $I_2$  are, respectively, the spatial coupling coefficients between the end plates and cavity, and  $\eta_c$  is the acoustical damping factor inside the cavity. Furthermore, since  $B_{1nm}^s$  and  $B_{2nm}^s$  can be obtained from the above equations, the behaviors of plate vibrations and the sound field under vibroacoustic coupling can be determined.

### 3. Theoretical results and discussion

#### 3.1 Effect of plate vibrations on acoustic characteristics

In the present study, the plates are assumed to be aluminum having a Young's modulus  $E$  of 71 GPa and a Poisson's ratio  $\nu$  of 0.33. Most of the plates have a radius  $a$  of 100, 150 or 200 mm and a thickness  $h$  of 3 mm. All combinations of these dimensions are considered in the analysis, however complementary dimensions are also used in the parametric study. The length of the cylindrical sound field having the same radius as the plates varies in length from 100 to 2000 mm. The support conditions of the plates, which have flexural rigidity  $D [= Eh^3/\{12(1 - \nu^2)\}]$ , are expressed by the non-dimensional stiffness parameters  $T_n$  ( $= T_1 a^3/D = T_2 a^3/D$ ) and  $R_n$  ( $= R_1 a/D = R_2 a/D$ ). In the present study, these values are identical for both plates. Here,  $R_n$  ranges from 0 to  $10^8$  when  $T_n$  is  $10^8$ , and hence the support condition varies from a simple support to a clamped support. The natural frequency of the plate corresponding to the  $(n, m)$  mode is expressed as  $f_{nm}$  and is regarded as the excitation frequency. This natural frequency depends both on the support conditions and the

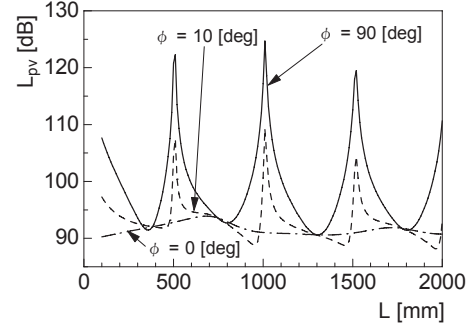


Fig. 2 Variation in sound pressure level with cylinder length in the (0,0) mode

vibration mode. When the (0,0), (1,0), and (0,1) modes are taken into consideration and the excitation position moves radially to a location near the nodal line or circle of the plate vibration, the excitation position also affects the plate vibration characteristics, the internal sound field, and the coupling between the respective vibrations and the sound field. The (1,0) mode has a nodal line on the radius, so that the excitation position is set as  $r_1/a = 0.4$  and remains distant from the nodal line. The behavior of the plate vibration of the (0,0) mode is only slightly influenced by varying the excitation position radially because this mode does not have a nodal line and a circle. Therefore,  $r_1/a = 0.4$  is used for comparison with the case of the (1,0) mode. On the other hand, a point force is applied to plate 1 at  $r_1/a = 0.2$  in the case of the (0,1) mode, so that the excitation position remains distant from the nodal circle of the (0,1) mode. However, point force  $F_1$  is set to 1 N in all modes. The resonance frequency of the cylindrical sound field is represented by  $f_{npq}$  [i.e., the natural frequency corresponding to the  $(n, p, q)$  mode].

Figure 2 shows the variations in the sound pressure level  $L_{pv}$  averaged over the entire sound field when the plate for which  $a = 150$  mm and  $h = 3$  mm, the  $R_n$  of which is  $10^8$  (i.e., the clamped support), is excited by the natural frequency of  $f_{00} = 340$  Hz and the cylinder length  $L$  is varied between 100 and 2000 mm. As mentioned in the previous section, the phase  $\phi_1$  of the plate 1 vibration is fixed as 0 deg, whereas the phase  $\phi_2$  of the plate 2 vibration is set to 0, 10, or 90 deg. These phases are related by the phase difference  $\phi$  as follows:

$$\phi = \phi_2 - \phi_1. \quad (9)$$

The sound pressure level  $L_{pv}$  varies only slightly over the entire range of  $L$  when  $\phi = 0$  deg but varies substantially and exhibits peaks in the vicinity of  $L = 510, 1010,$  and  $1520$  mm when  $\phi = 10$  and  $90$  deg. The value of  $L_{pv}$  is lower when  $\phi = 10$  and  $90$  deg and is almost identical at all phase differences near  $L = 380, 750, 1290,$  and  $1770$  mm. The

acoustic modes that cause  $L_{pv}$  to have peaks at  $L = 510, 1010,$  and  $1520$  mm are conjectured to be  $(0,0,q)$  modes because both plates vibrate in the  $(0,0)$  mode.

The vibration of plate 2 has a significant effect on the formation of the sound field and vibroacoustic coupling, despite not being driven by a harmonic point force, whereas the influence of  $\phi$  on the acoustic characteristics has only been described for  $\phi = 0, 10,$  and  $90$  deg. Figure 3 shows the variation in  $L_{pv}$  when  $\phi$  ranges from  $0$  to  $180$  deg. In order to clearly indicate changes in  $L_{pv}$  with respect to  $L$ , the range of  $L$  is selected to be between  $830$  and  $1010$  mm because minimum and maximum values, respectively, of  $L_{pv}$  are near these lengths when  $\phi = 90$  deg, as shown in Fig. 2. Note that  $L_{pv}$  exhibits minimum and maximum values in the vicinity of  $\phi = 40$  and  $130$  deg, respectively, at  $L = 830$  mm. These minimum and maximum values shift to lower  $\phi$  as  $L$  increases, and the maximum value of  $L_{pv}$  increases as  $\phi$  decreases. When  $L = 1010$  mm, the maximum value of  $L_{pv}$  appears in the vicinity of  $\phi = 90$  deg, whereas the minimum value of  $L_{pv}$  disappears from the range of  $\phi$ . This change in  $L_{pv}$  with  $\phi$  indicates that the acoustic characteristics depend strongly on the vibration of plate 2, i.e., there are ranges of  $\phi$  that intensify or suppress coupling between the plate vibrations and the sound field. The values of  $\phi$  at which  $L_{pv}$  is a maximum are denoted by  $\phi_{max}$ .

**3.2 Phase difference that maximizes the sound pressure level**

Figure 4(a) shows the variations in  $\phi_{max}$  for both the  $(0,0)$  and  $(1,0)$  modes in order to clarify the difference between these two vibration modes, and the values of  $\phi_{max}$  that maximize  $L_{pv}$  are plotted by circles. For the  $(0,0)$  mode,  $\phi_{max}$  is approximately  $83$  deg at  $L = 100$  mm and decreases gradually with increasing  $L$  up to approximately  $L = 380$  mm, where  $\phi_{max}$  suddenly increases to over  $90$  deg and then decreases with increasing  $L$  again. This behavior of  $\phi_{max}$  is repeated in a similar manner as  $L$  increases to  $L = 2000$  mm. The abrupt changes in  $\phi_{max}$  are centered at approximately  $90$  deg. When plate 1 vibrates in the  $(1,0)$  mode, for which

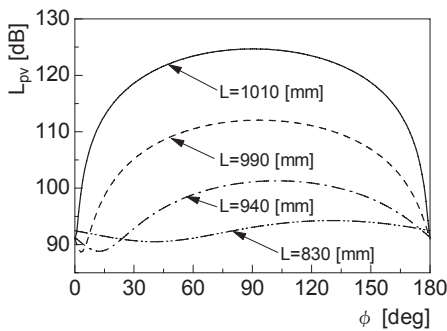
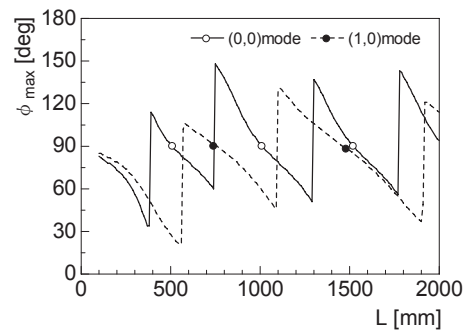


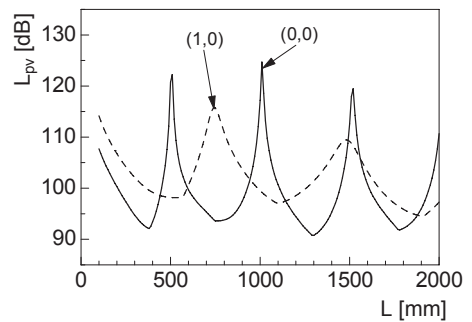
Fig. 3 Variation in sound pressure level with phase difference in the  $(0,0)$  mode

$f_{10}$  is  $710$  Hz, although the rapid changes in  $\phi_{max}$  differ from those of the  $(0,0)$  mode in  $L$ ,  $\phi_{max}$  behaves in a similar manner to that in the  $(0,0)$  mode. Figure 4(b) shows the variations in  $L_{pv}$  corresponding to the variation in  $\phi_{max}$  with  $L$ . Here,  $L_{pv}$  corresponding to  $\phi_{max}$  has maxima as  $\phi_{max}$  is approximately  $90$  deg and is denoted by the circles in Fig. 4(a). Peaks in  $L_{pv}$  indicate that vibroacoustic coupling between the plate vibrations and the sound field is promoted. Note that  $f_{nm}$  and  $f_{npq}$  must be approximately equal for peaks in  $L_{pv}$  to appear.

Figures 5(a) and 5(b), respectively, show plots of  $\phi_{max}$  and  $L_{pv}$  for the plates of  $h = 3$  mm and  $R_n = 10^8$  with  $a = 100$  and  $200$  mm that vibrate in the  $(1,0)$  mode, for which  $f_{10}$  is  $1595$  and  $400$  Hz, respectively. The periods of  $\phi_{max}$  and the peaks of  $L_{pv}$  at  $a = 100$  mm become shorter than those at  $a = 150$  mm because the natural frequency increases with decreasing  $a$ . However, when  $a = 200$  mm, the behaviors of  $\phi_{max}$  and  $L_{pv}$ , being non-periodic, differ significantly from the above results. Note that  $\phi_{max}$  remains almost constant in the vicinity of  $90$  deg and decreases abruptly around  $L = 430, 860, 1290,$  and  $1720$  mm. In contrast,  $L_{pv}$  tends to decrease in a steadier fashion with increasing  $L$  and has extremely small peaks when  $\phi_{max}$  decreases abruptly. The plate vibration of the  $(1,0)$  mode is also coupled to the sound field of the  $(1,1,q)$  modes due to their similarity to the acoustic mode. Consequently, each acoustic mode exhibits peaks in  $L_{pv}$  with



(a) Phase difference



(b) Sound pressure level

Fig. 4 Variations in phase difference and sound pressure level with cylinder length in the  $(0,0)$  and  $(1,0)$  modes

strong coupling with the respective vibration modes, when the natural frequency  $f_{10}$  of the vibration mode coincides with the resonance frequency  $f_{11q}$  of the corresponding acoustic mode at  $a = 100$  and  $150$  mm. However, for the case in which  $a = 200$  mm for the  $(1,0)$  mode,  $f_{10}$  never reaches  $f_{11q}$  over the entire range of  $L$  and decreases with increasing  $a$ , so that  $\phi_{max}$  does not exhibit abrupt changes, which indicate shifts in  $q$  of the  $(1,1,q)$  mode, and  $L_{pv}$  does not peak decidedly, as in the other cases, because coupling between  $(1,0)$  and  $(1,1,q)$  modes is suppressed due to the relationship between  $f_{10}$  and  $f_{11q}$ . In contrast with the  $(1,1,q)$  modes, the  $(0,0,q)$  modes give rise to peaks in  $L_{pv}$ , but these peaks are very small due to the significant difference between the  $(1,0)$  and  $(0,0,q)$  modes in modal shapes.

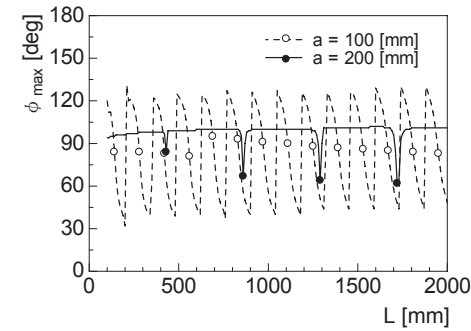
Figures 6(a) and 6(b), respectively, show  $\phi_{max}$  and  $L_{pv}$  as functions of  $L$  for the plate with  $a = 150$  mm,  $h = 3$  mm, and  $R_n = 10^8$ , which vibrates in the  $(0,1)$  mode. Note that  $\phi_{max}$  varies with a short period and behaves in a more complicated manner as compared to the other cases. In contrast,  $L_{pv}$  has peaks at intervals of approximately 130 mm and has more peaks over the entire range of  $L$  than other cases, even though  $f_{01} = 1325$  Hz, which never reaches  $f_{01q}$  over the entire range of  $L$ . The peaks are caused by the  $(0,0,q)$  modes, the resonance frequencies of which are equal to  $f_{01}$ , because, like the  $(0,0)$

mode, the  $(0,1)$  mode, being distinct from the two-dimensional  $(0,0,q)$  modes in a lateral cross-sectional plane of the cavity in the radial order, is symmetric about the  $z$ -axis. The  $(0,0,q)$  modes are the dominant modes for all values of  $L$ .

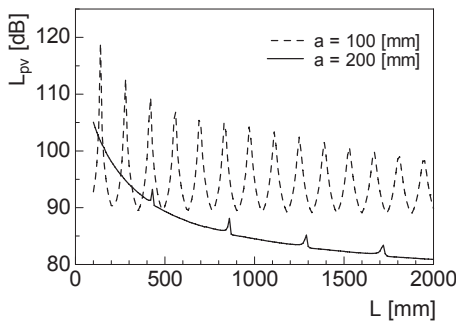
### 3.3 Parametric study of conditions to enhance vibroacoustic coupling

In order to clarify the geometric conditions under which vibroacoustic coupling is promoted, the geometric parameters  $L_{qh}$  and  $a_h$  are defined. When the natural frequency  $f_{nm}$  is coincident with the resonance frequency  $f_{npq}$ , i.e.,  $L_{pv}$  peaks, as shown in the previous section,  $L_{qh}$  is the length  $L$  divided by the longitudinal order  $q$  and the thickness  $h$ , and  $a_h$  is the ratio of the radius  $a$  to  $h$ .

Figure 7(a) shows the relationship between  $L_{qh}$  and  $a_h$  for the  $(0,0)$ ,  $(1,0)$ , and  $(0,1)$  modes when  $R_n = 10^8$ . The lines for the  $(0,0)$ ,  $(1,0)$ , and  $(0,1)$  modes are plotted based on  $f_{00q}$ ,  $f_{11q}$ , and  $f_{01q}$  respectively, the acoustic modes of which have similar modal shapes to their vibration modes. The figure also shows the lines for the  $(1,0)$  and  $(0,1)$  modes based on  $f_{00q}$  due to coupling with  $(0,0,q)$  modes.  $L_{qh}$  tends to increase with  $a_h$ . In particular,  $L_{qh}$  based on  $f_{00q}$  is linear over the entire range of  $a_h$ . On the other hand, when  $a_h$  is varied in the proximity of each  $L_{qh}$  based on  $f_{00q}$  in the range of small  $a_h$ ,  $L_{qh}$  based

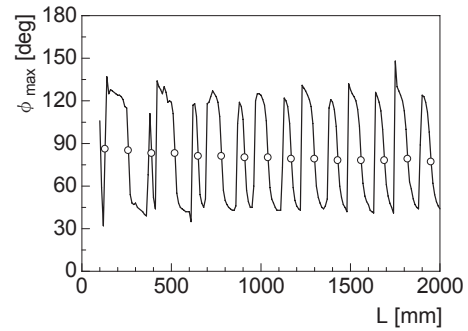


(a) Phase difference

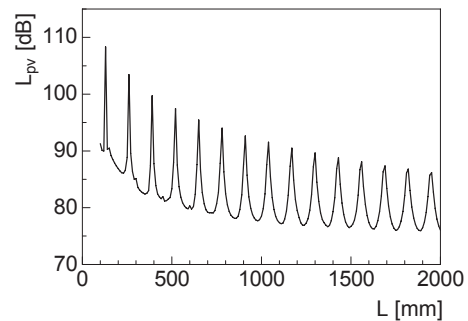


(b) Sound pressure level

Fig. 5 Variations in phase difference and sound pressure level with cylinder length in the  $(1,0)$  mode when plate radius changes



(a) Phase difference

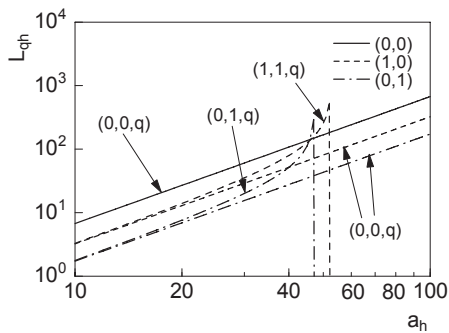


(b) Sound pressure level

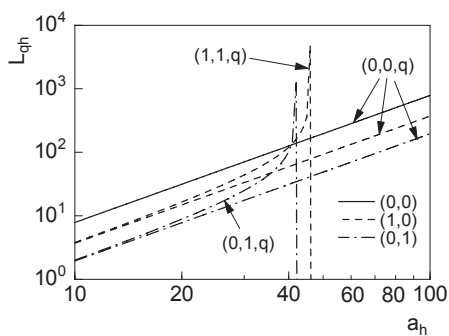
Fig. 6 Variations in phase difference and sound pressure level with cylinder length in the  $(0,1)$  mode

on  $f_{11q}$  and  $f_{01q}$  have higher rates of increase than  $L_{qh}$  based on  $f_{00q}$  and the difference between these  $L_{qh}$  increases gradually with increasing  $a_h$ . The difference between  $L_{qh}$  based on  $f_{11q}$  and  $f_{01q}$  and  $L_{qh}$  based on  $f_{00q}$  continues to increase up to  $a_h = 52$  and  $47$ , respectively, and  $L_{qh}$  based on  $f_{11q}$  and  $f_{01q}$  does not exist beyond these points. Figure 7(b) shows the relationship between  $L_{qh}$  and  $a_h$  when  $R_n = 10$ . The changes in  $L_{qh}$  with  $a_h$  are similar to those for  $R_n = 10^8$ , whereas the variation of  $L_{qh}$  with  $a_h$  differs from that of  $R_n = 10^8$  when  $L_{qh}$  vanishes.

As described above, the condition for enhancing vibroacoustic coupling and for maximizing the sound pressure level can be determined from  $a_h$  and  $L_{qh}$  according to their variation with the vibration mode and the support conditions of the plate. Applied to design of the new electricity generation system, the results reveal that the (0,0) and (0,1) modes can cover a wide range of  $a_h$ . However, in the (0,1) mode, the (0,0,q) mode is excited as the dominant acoustic mode for almost all  $a_h$ , because the (0,0,q) mode is similar to the (0,1) mode from the viewpoint of the symmetry about the  $z$ -axis, and the natural frequency of the (0,1) mode can reach its resonance frequency, which is always lower than the natural frequency of the (0,1,q) mode. On the other hand, in the (1,0) mode, the application is limited by  $a_h$ , because the (1,1,q) mode that promotes coupling is extremely suppressed under the condition that  $f_{10}$  cannot reach  $f_{11q}$ .



(a)  $R_n = 10^8$



(b)  $R_n = 10$

Fig. 7 Conditions by which to maximize the sound pressure level in expressions involving the terms  $L_{qh}$  and  $a_h$

## 4. Experiment

### 4.1 Experimental apparatus and method

Figure 8 shows the experimental apparatus used in the present study. The cylindrical structure consists of a steel cylinder with circular aluminum end plates that are 3 mm thick. The cylinder has inner radius of 153 mm, and these lengths can be varied from 200 to 2000 mm in order to emulate the analytical model. A harmonic point force excites only one end plate. This force is applied to the plate by a small vibrator, the amplitude of which is controlled to be 1 N. The excitation frequencies  $f_e$  are the natural frequencies  $f_{nm}$  obtained from the experimental modal analysis. The position of the point force  $r_1$  is normalized by radius  $a$ , and the (0,0), (1,0), and (0,1) modes are used in this investigation. The position of the point force is set to  $r_1/a = 0.4$  for the (0,0) and (1,0) modes and to  $r_1/a = 0.2$  for the (0,1) mode in order to avoid the nodal circle. In the present study, the main characteristic of the plate vibration under consideration is the phase difference between the plate vibrations. Therefore, acceleration sensors are installed on both plates to measure this phase difference. In order to estimate the internal acoustic characteristics, the sound pressure level in the cavity is measured by using a condenser microphone with a probe tube, the tip of which is located in the vicinity of the non-excited plate and the cylinder wall, which is the approximate location of the maximum sound pressure level.

### 4.2 Experimental results and discussion

Figure 9(a) shows the variations in the theoretical and experimental phase differences between the vibrations of the two plates as a function of cylinder length  $L$ . The excitation frequency  $f_e$  is set to 280 Hz in order to induce the (0,0) mode for a plate in this analysis. The experimental value of  $f_e$  is 282 Hz. In the theoretical results, only  $\phi_{max}$ , at which the

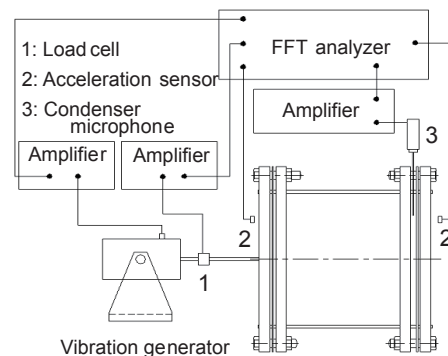
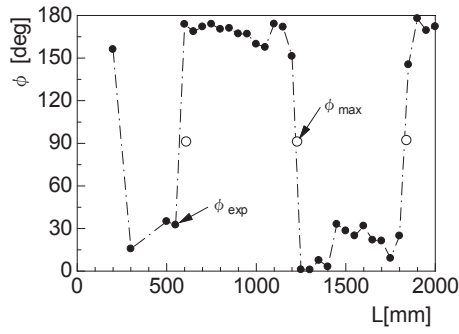
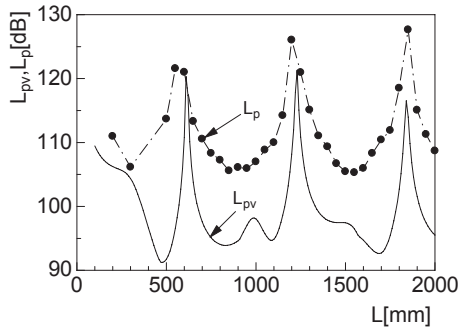


Fig. 8 Configuration of the experimental apparatus



(a) Phase difference

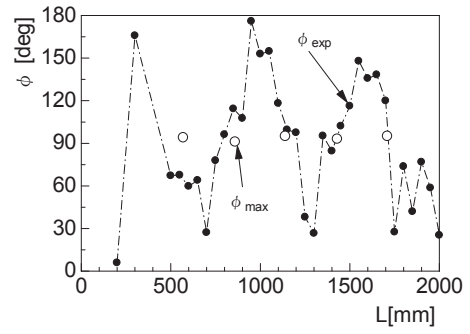


(b) Sound pressure level

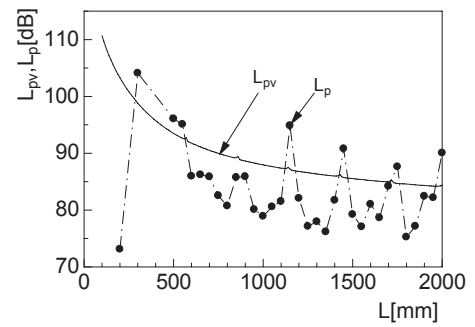
Fig. 9 Comparison between theoretical and experimental results for phase difference and sound pressure level in the (0,0) mode

sound pressure level  $L_{pv}$  averaged over the entire sound field peaks, is plotted in order to indicate the amplification of the sound pressure level. The theoretical results for  $\phi_{max}$  differ from those for  $a = 150$  mm in Fig. 4(a), in which the intervals between the values of  $L$  for the above  $\phi_{max}$  are shortened, because the natural frequency decreases with the decrease in  $R_n$ . In the experimental results, the value of  $\phi_{exp}$  remains approximately constant at 180 deg when  $L$  is extended from 550 to 1150 mm, but falls rapidly to approximately 0 deg and remains close to 0 deg for values of  $L$  up to 1750 mm. Beyond  $L = 1750$  mm,  $\phi_{exp}$  again increases rapidly to approximately 180 deg. The phase difference then remains almost constant up to  $L = 2000$  mm. Figure 9(b) shows the variations in  $L_{pv}$ , corresponding to  $\phi_{max}$  in the analysis, and in the sound pressure level  $L_p$  measured in the experiment. Peaks in  $L_{pv}$  for  $\phi_{max}$  appear at  $L = 610, 1230,$  and  $1840$  mm, at which  $\phi_{max}$  is approximately 90 deg. These peaks are caused by the (0,0,1), (0,0,2), and (0,0,3) modes, respectively. Here,  $L_p$  increases significantly at 550, 1200, and 1850 mm. In addition,  $\phi_{exp}$  exhibits rapid changes around these values of  $L$ , and the  $\phi_{max}$  appears at the midpoints of the rapid changes in  $\phi_{exp}$ .

Figures 10(a) and 10(b) show the theoretical and



(a) Phase difference



(b) Sound pressure level

Fig. 10 Comparison between theoretical and experimental results for phase difference and sound pressure level in the (1,0) mode

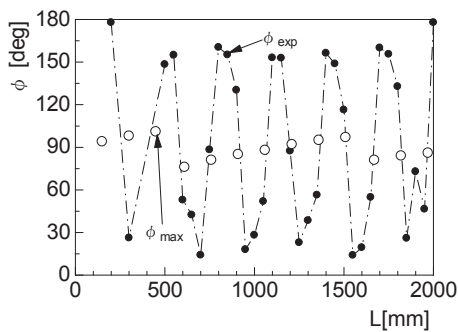
experimental results for the phase difference and sound pressure level for the (1,0) mode when a plate is excited at  $f_e = 600$  Hz. Figure 10(a) shows the variations in  $\phi_{max}$  and  $\phi_{exp}$ , and Fig. 10(b) shows the variations in  $L_{pv}$  and  $L_p$  with  $L$ . In this case,  $f_e$  does not reach  $f_{11q}$  because  $a_h$  is greater than 50 [see Fig. 7(b)], so that the (1,0) mode is poorly coupled with the (1,1,q) modes. Since the (1,0) mode is coupled to the (0,0,q) modes that differ from the two-dimensional modes,  $\phi_{max}$  is approximately 90 deg in the vicinity of  $L = 850, 1150, 1450,$  and  $1750$  mm. The experimentally measured  $\phi_{exp}$  varies frequently between 0 deg and 180 deg. Being somewhat erratic, these changes are gradual in comparison with those of the (0,0) mode. Therefore,  $L_{pv}$  does not have peaks like those for the (0,0) mode, and  $L_{pv}$  decreases monotonically with increasing  $L$ , as in the case for  $a = 200$  mm, as shown in Fig. 5(b). In addition,  $L_{pv}$  has extremely small peaks in the vicinity of the above values of  $L$ . In contrast, although  $L_p$  tends to decrease with increasing  $L$  in a similar manner to  $L_{pv}$ , the peaks in  $L_p$  are clearer than the peaks in  $L_{pv}$  in the vicinity of the above values of  $L$ . Moreover,  $\phi_{exp}$  approaches 90 deg when  $L_{pv}$  and  $L_p$  peak at the above values of  $L$ , so that the behavior of  $\phi_{exp}$  associated with the maximum sound pressure level is similar to that of the (0,0) mode.

In order to theoretically and experimentally investigate the (0,1) mode, when a point force of  $f_e = 1135$  Hz is applied to a plate, Fig. 11(a) shows the variations in  $\phi_{max}$  and  $\phi_{exp}$  with  $L$  and Fig. 11(b) shows the variations in  $L_{pv}$  and  $L_p$  with  $L$ . In this case,  $f_e$  never reaches  $f_{01q}$  because  $a_h$  is greater than 50 [see Fig. 7(b)], so that the (0,1) mode is coupled with the (0,0, $q$ ) modes instead of with the (0,1, $q$ ) modes. Since both the (0,1) and (0,0, $q$ ) modes are symmetric about the  $z$ -axis, even though these modes do not coincide on the radial order  $m$ ,  $\phi_{max}$  appears at the interval of the (0,0, $q$ ) mode due to coupling between these modes. Note that  $\phi_{exp}$  varies through the circles of  $\phi_{max}$  in the vicinity of 90 deg in a somewhat erratic manner. These results are based on relatively rough measurements, as compared to the measurements for the other modes, due to shortening of the periodic intervals. However, both  $L_{pv}$  and  $L_p$  have peaks in the vicinity of 90 deg, around which  $\phi_{exp}$  approaches  $\phi_{max}$ .

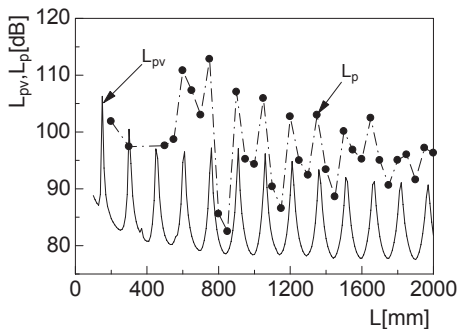
5. Conclusion

In order to apply the acoustic energy to the new electricity generation system, vibroacoustic coupling between plate vibrations and a sound field is investigated for a cylindrical structure with circular end plates. Only one end plate is excited by a harmonic point force, the frequency of which is the natural frequency of each plate. Vibroacoustic coupling is estimated for different radii, thicknesses, and support conditions of the plate. The present study focused on the phase difference between the vibrations of the two plates in order to promote coupling and to maximize the sound pressure level.

The theoretical investigation revealed that if the natural frequency reaches the resonance frequency, the two-dimensional mode of which has a similar modal shape to the vibration mode, the sound pressure level is maximized when the phase difference is close to 90 deg. By using the cylinder length and the radius and thickness of the plates as geometric parameters, the relationships between the parameters can be completely normalized for each vibration mode and each support condition, enabling the dimensions that maximize the sound pressure level via vibroacoustic coupling to be predicted. The experiment results demonstrate that the theoretical estimation is justified via changing the cylindrical length and vibration modes, so that the theoretical results are regarded as a significant reference for the design of a new electricity generation system.



(a) Phase difference



(b) Sound pressure level

Fig. 11 Comparison between theoretical and experimental results for phase difference and sound pressure level in the (0,1) mode

6. References

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