

# Free Vibration Analysis of a Model Structure with New Tuned Cradle Mass Damper

by

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(Received on Mar. 24, 2012 and accepted on May 17, 2012)

## Abstract

Tuned cradle mass dampers (TCMDs) use the motion of a swing mass on a curved surface to dissipate structural vibration energy. In this study, we developed a TCMD that had a constant swing speed. We verify its performance by performing experiments and numerical analysis for a structure undergoing free vibrations. To obtain a device with a constant speed, variable radii of the curved surfaces were calculated using simple pendulum dynamics. In this study, the damper was installed in a simple one-story rigid-frame model with a frequency of approximately 1 Hz and free vibrations was excited. The numerical analysis results agreed well with the experimental results.

**Keywords:** tuned mass damper, free vibration, structural control

## 1. Introduction

Vibrations are excited in structures by external dynamic forces generated by natural phenomena such as earthquakes and wind and human activities such as traffic, civil engineering work, and construction work. A structure generally exhibits low damping when its natural vibration modes are excited by an external dynamic force. Resonance may cause structural damage, which can potentially lead to significant loss of life and reduced liveability. Vibration control systems have recently been applied to overcome these vibration problems. Vibration control systems can be broadly classified into two types: active systems that require an external energy source to absorb vibrational energy and passive systems that do not require an energy source. Several kinds of practical passive controllers have been developed; these include a controller that uses laminated rubber or a coil spring to support the weight of a damper, a suspended damper that employs the principle of a pendulum<sup>1)</sup>, an impact damper that uses a steel ball<sup>2)</sup>, a tuned liquid damper that uses liquids<sup>3-5)</sup>, and a tuned rotary-mass damper<sup>6,7)</sup> consisting of a rolling mass and a container that permits the mass to move freely along its inner arc.

The tuned cradle mass damper (TCMD)<sup>8)</sup> is another passive damper. It relies on the movement of the swing

mass along a curved surface to dissipate the vibrational energy of a structure. TCMDs have the advantages of being simple, compact, and easy to maintain. The swing mass has small wheels attached that allow the mass to move along the curved surface. A recently developed TCMD employs a rail with a variable radius curve to realize a constant swing speed for large oscillations. Simple pendulum dynamics is used to determine the variable radius curve.

This study develops a model of the new TCMD and verifies its performance through both experiments and numerical analysis.

## 2. Device configuration

### 2.1 Experimental model

A model of a one-story structure is used to experimentally examine the TCMD. Figure 1 shows the model. It is made of steel PL-60×8 (SS400) and uses four 1300-mm-high columns to support a floor. The lateral spring constant of the structure,  $k_1$ , is 11.1 kgf/cm. Figure 2 shows the force–displacement relationship of the structure. The structural model has an effective mass of 220 kgf (including the TCMD) and a natural frequency of approximately 1.12 Hz. The diamond symbols (◆) in experimental waveform are used to calculate the damping constant  $h_1$  ( $= 0.0013$ ) of the structure.

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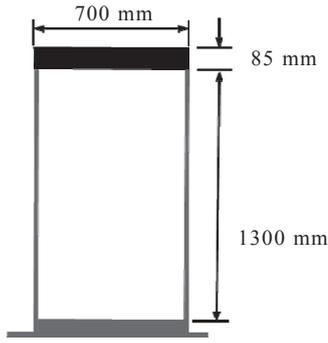


Fig. 1 Structure

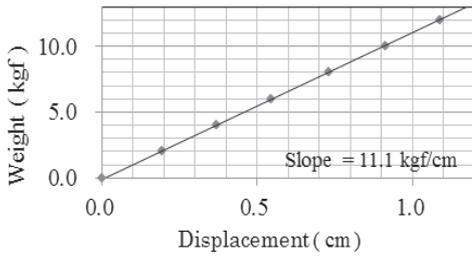


Fig. 2 Force-displacement line of the structure

### 2.2 Modeling a cradle TCMD

Figure 3 shows plan, front, and side views of the TCMD model used in the experiments. The TCMD is made of three copper plates. The swing mass has three wheels. The dimensions of the middle plate are 500×280×5 mm and those of the side plates are 710×280×5 mm. The wheel has a diameter of 72 mm. The swing mass (including the wheels) is about 6 kg. The mass moves along three curved surfaces as the structure moves. To realize magnetic damping, neodymium magnets are attached to both sides of the swing mass. A magnetic field is then generated when the mass moves along the copper plates. The damping strength of the mass can be adjusted by varying the number of magnets attached to the swing mass.

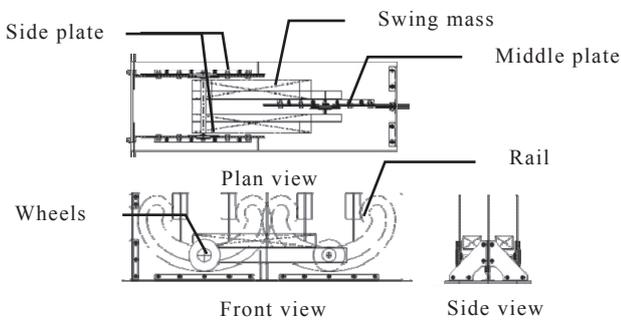


Fig. 3 TCMD

Figure 4 shows the relationship between the damping constant  $h_2$  and the number of magnets used for the TCMD.

Here,  $h_2$  is the damping constant obtained from free vibrations of the TCMD. The TCMD has a natural frequency of 1.19 Hz; it is constant for large amplitudes of the mass swing.

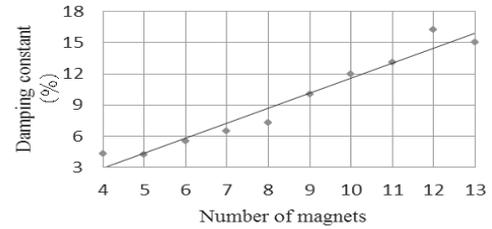


Fig. 4 Plot of damping constant ( $h_2$ ) against number of magnets

## 3. Numerical analysis

### 3.1 Determination of constant swing mass speed

The equation of motion for a simple pendulum with no damping force is given by:

$$\frac{d^2\theta}{dt^2} + \frac{g}{\ell_p} \sin\theta = 0 \quad (1)$$

where  $g$  is the acceleration due to gravity,  $\ell_p$  is the pendulum length,  $t$  is the time and  $\theta$  is the angular displacement. The period  $T_p$  for the motion described in

Eq. (1) is given by: 
$$T_p = 4\sqrt{\frac{\ell_p}{g}} K \quad (2)$$

where 
$$K = \int_0^{\pi/2} \frac{d\phi}{\sqrt{1 - \sin^2(\theta_m/2) \sin^2\phi}} \quad (3)$$

In Eq. (3),  $\theta_m$  is the maximum swing angle of the pendulum. To obtain a constant  $T_p$  for large oscillation

amplitudes, Eq. (2) can be rewritten for  $\ell_p$  as

$$\ell_p = \left(\frac{T_p}{4K}\right)^2 \cdot g \quad (4)$$

We then calculate the variable radius  $\ell_p$  for each  $\theta_m$ . Simpson's rule is used to integrate Eq. (3). To obtain a constant speed for the TCMD, a variable radius  $\ell_p$  is used to design a curved rail surface.

### 3.2 Numerical analysis of model structure and TCMD

Figure 5 shows a schematic diagram of the swing mass.

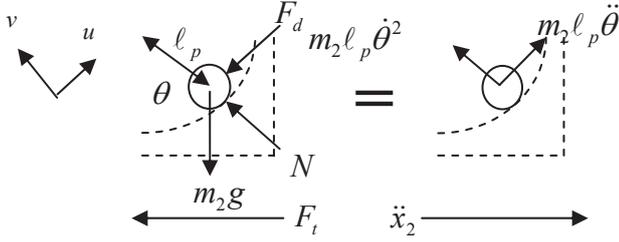


Fig.5 Free body diagram of TCMD

Here,  $m_2$  is the mass of the swing. Because the mass of the wheels is small relative to the swing mass, alternatively, we ignore the rotational energy of the wheels. In addition,  $F_t$  is the horizontal force given by the TCMD,  $\ddot{x}_2$  is the horizontal acceleration supplied by the structure,  $F_d$  is the damping force, and  $N$  is the normal force. The equation of motion for mass  $m_2$  in the  $u$  and  $v$  directions can be written as

$$u \text{ direction: } -m_2 g \cdot \sin \theta - F_d = m_2 \ell_p \ddot{\theta} \quad (5)$$

$$v \text{ direction: } -m_2 g \cdot \cos \theta + N = m_2 \ell_p \dot{\theta}^2 \quad (6)$$

$$\text{where } F_d = c_2 \ell_p \dot{\theta} \quad (7)$$

only viscous damping  $c_2$  is considered here; for simplicity, we ignore frictional damping between the wheel and the curved surface. The equation of motion for the cradle in the horizontal direction then becomes

$$m_2 \ddot{x}_2 + F_t = 0 \quad (8)$$

$$\text{where } F_t = N \cdot \sin \theta + F_d \cdot \cos \theta \quad (9)$$

$$\text{or } \ddot{x}_2 = -\ell_p \dot{\theta}^2 \cdot \sin \theta - \frac{1}{m_2} c_2 \ell_p \dot{\theta} \cdot \cos \theta - g \cdot \cos \theta \sin \theta \quad (10)$$

Figure 6 shows a schematic diagram of the structure.

Here,  $m_1$  is the mass of the structure,  $c_1$  is the viscous damping coefficient,  $k_1$  is the spring constant of the structure, and  $x_1$  is the horizontal displacement of the

structure. We then obtain the following equation of motion for the structure.

$$-F_t + c_1 \dot{x}_1 + k_1 x_1 = -m_1 \ddot{x}_1 \quad (11)$$

$$\text{or } \ddot{x}_1 = -\frac{c_1}{m_1} \dot{x}_1 - \frac{k_1}{m_1} x_1 + \frac{m_2}{m_1} \ell_p \dot{\theta}^2 \cdot \sin \theta + \frac{m_2}{m_1} g \cdot \cos \theta \sin \theta + \frac{c_2 \ell_p \dot{\theta}}{m_1} \cos \theta \quad (12)$$

We can now numerically calculate the coupled equations (10) and (12).



Fig.6 Free body diagram of structure

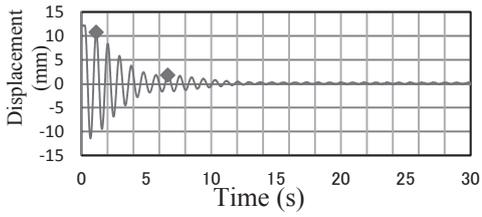
## 4. Experimental results

Experimental measurements were performed to clarify the dissipation of the vibrational energy of the simple model structure by the TCMD. Laser displacement sensors were used to measure the time-displacement responses of the structure and the TCMD. The measurement sampling rate was 5 ms. An initial lateral displacement of 12 mm was given to the structure. Beats are observed because of the weak damping of the TCMD.

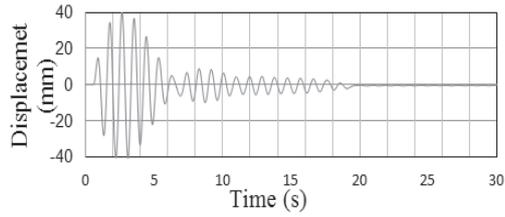
When the swing mass with magnets moves on the curved surface, the structural damping becomes much greater than that without the magnets. Figures 8–10 show the examples of experimental and analytical waveforms for the structure and TCMD; Figures 7-9 (a) and (b) show the experimental waveforms obtained when six, nine, and 13 magnets were used for the TCMD, respectively.

## 5. Analysis results

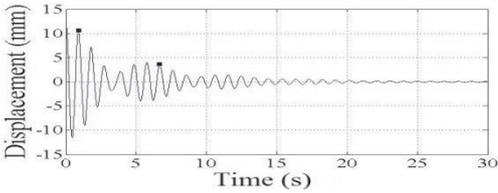
The fourth-order Runge-Kutta method was used to numerically solve Eqs. (10) and (12);  $\ell_p$  was updated in each iteration time step. Figures 7-9 compare the experimental and analysis results for the structure under free vibrations.



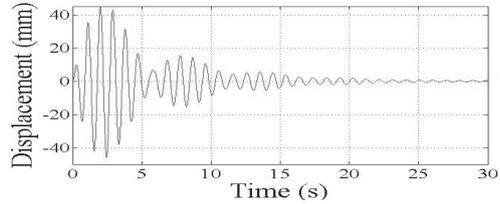
a) Structure responses (experiment)



b) TCMD responses (experiment)

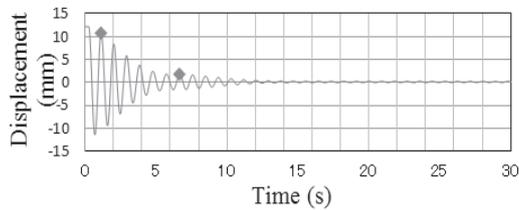


c) Structure responses (analysis)

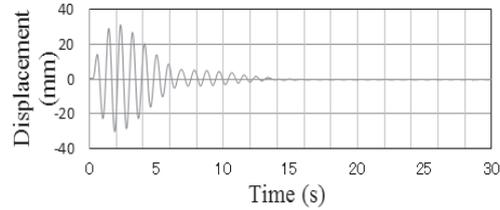


d) TCMD responses (analysis)

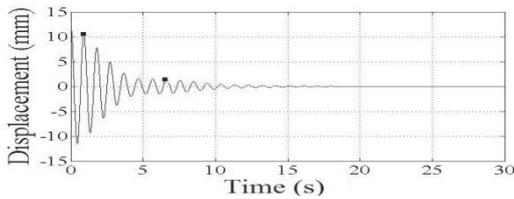
Fig.7 Displacement responses for  $h_2 = 5.48\%$  (six magnets)



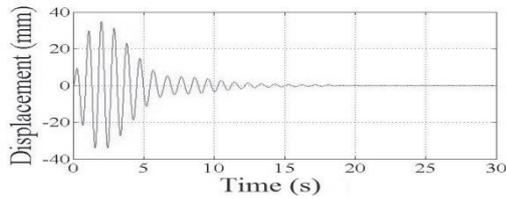
a) Structure responses (experiment)



b) TCMD responses (experiment)

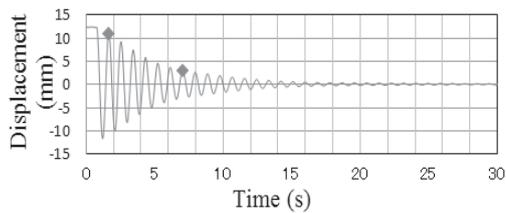


c) Structure responses (analysis)

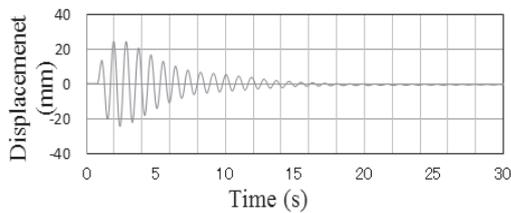


d) TCMD responses (analysis)

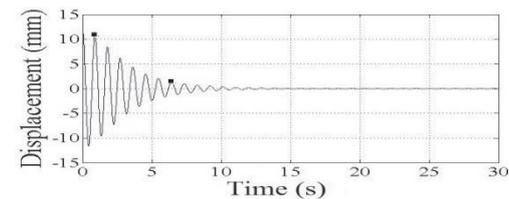
Fig. 8 Displacement responses for  $h_2 = 10.05\%$  (nine magnets)



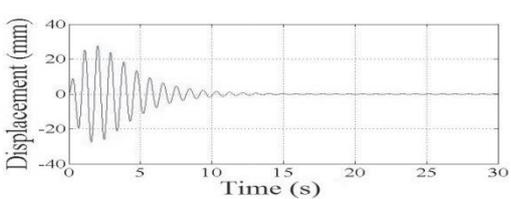
a) Structure responses (experiment)



b) TCMD responses (experiment)



c) Structure responses (analysis)



d) TCMD responses (analysis)

Fig.9 Displacement responses for  $h_2 = 15.02\%$  (13 magnets)

Figure 10 shows a plot of the experimentally obtained viscous damping constant  $h_1$  of the structure obtained against the number of magnets. The vertical axis represents  $h_1$  and the horizontal axis represents the number of magnets used in the TCMD. The heights of the two points indicated by the diamond symbols in the Figures (a) are used to calculate  $h_1$ . Figure 11 shows the viscous damping constant  $h_1$  of the structure obtained from the numerical analysis. The vertical axis represents  $h_1$  and horizontal axis represents the damping constant  $h_2$

from

Fig.4. The damping constant of the structure ( $h_1$ ) obtained by the experiment agrees well with those obtained by analysis.

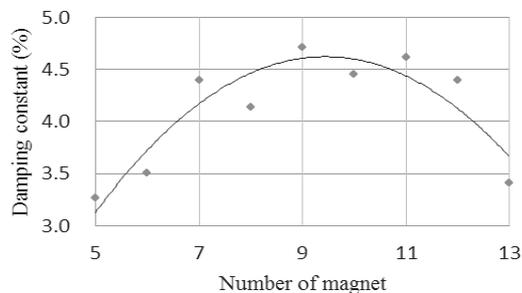


Fig.10 Damping constant versus number of magnets (experiment)

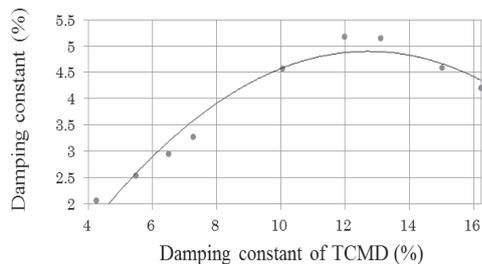


Fig.11 Damping constant versus damping ratio of TCMD (analysis)

## 6. Conclusion

This study proposed a new vibration damping device (TCMD) that consists of a swing mass and a curved cradle with a variable radius. A constant swing mass speed was attained using a cradle designed by a simple pendulum dynamics. The performance of the new TCMD was demonstrated through experiments and numerical analysis for a structure with a frequency of approximately 1 Hz under free vibrations. The new TCMD is more compact than conventional devices. In the future, we intend to determine the optimal parameters using the numerical model developed in this study.

## References

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